Primitives - Intégrales

2.2.6 Calculer :

a) $\int \cos^2(x)\,dx$ c) $\int \cos^3(x)\,dx$

b) $\int \sin^2(x)\,dx$ d) $\int \sin^4(x)\,dx$

a) $\int \cos^2(x)\,dx = \int \frac{1+\cos(2x)}{2}\,dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)\,dx$

$= \frac{1}{2}\int dx + \frac{1}{2}\cdot\frac{1}{2}\int \underbrace{\cos(2x)}_{\cos(u)}\cdot\underbrace{2}_{u'}\,dx = \boxed{\frac{x}{2} + \frac{\sin(2x)}{4} + C}$

b) $\int \sin^2(x)\,dx = \int (1-\cos^2(x))\,dx = \int dx - \int \cos^2(x)\,dx$

$= x - \left(\frac{x}{2} + \frac{\sin(2x)}{4}\right) + C$ (question a)

$= x - \frac{x}{2} - \frac{\sin(2x)}{4} + C = \boxed{\frac{x}{2} - \frac{\sin(2x)}{4} + C}$

c) $\int \cos^3(x)\,dx = \int (1-\sin^2(x))\cos(x)\,dx = \int \left(\cos(x) - \cos(x)\sin^2(x)\right)dx$

$= \int \cos(x)\,dx - \int \underbrace{\sin^2(x)}_{u^2}\underbrace{\cos(x)}_{u'}\,dx = \boxed{\sin(x) - \frac{\sin^3(x)}{3} + C}$

d) $\int \sin^4(x)\,dx = \int \left(\frac{1-\cos^2(2x)}{2}\right)^2 dx = \int \left(\frac{1-2\cos(2x)+\cos^2(2x)}{4}\right)dx$

$= \frac{1}{4}\int (1-2\cos(2x))\,dx + \frac{1}{4}\int \frac{1+\cos(4x)}{2}\,dx$

$= \frac{1}{4}\int 1 - \underbrace{2}_{u'}\underbrace{\cos(2x)}_{\cos(u)}\,dx + \frac{1}{8}\int 1\,dx + \frac{1}{32}\int \underbrace{\cos(4x)}_{\cos(u)}\cdot\underbrace{4}_{u'}\,dx$

$$= \frac{x}{4} - \frac{\sin(2x)}{4} + \frac{x}{8} + \frac{\sin(4x)}{32} + C = \boxed{\frac{12x - 8\sin(2x) + \sin(4x)}{32} + C}$$

2.2.7 Calculer :

a) $\int \sin^2(x)\cos^2(x)\,dx$

b) $\int \sqrt{\sin(x)}\cos^3(x)\,dx$

c) $\int \sin(5x)\cos(3x)\,dx$

d) $\int \frac{\cos(x)}{2-\sin(x)}\,dx$

a) $\int \sin^2(x)\cos^2(x)\,dx = \int \left(\frac{1-\cos(2x)}{2} \cdot \frac{1+\cos(2x)}{2}\right) dx$

$= \int \frac{1-\cos^2(2x)}{4}\,dx = \int \frac{1}{4}\,dx - \int \frac{\cos^2(2x)}{4}\,dx$

$= \frac{1}{4}\int dx - \frac{1}{4}\int \frac{1+\cos(4x)}{2}\,dx = \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{8}\int \cos(4x)\,dx$

$= \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{8}\cdot\frac{1}{4}\int \underbrace{\cos(4x)}_{\cos(u)} \cdot \underbrace{4}_{u'}\,dx$

$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{32}\sin(4x) + C$

$= \boxed{\frac{1}{8}x - \frac{1}{32}\sin(4x) + C}$

b) $\int \sqrt{\sin(x)}\cos^3(x)\,dx = \int \sqrt{\sin(x)}\cdot\cos(x)\cdot\cos^2(x)\,dx$

$= \int \sqrt{\sin(x)}\cdot\cos(x)\cdot\left(1-\sin^2(x)\right)dx = \int \sqrt{\sin(x)}\cos(x)\,dx - \int \sqrt{\sin(x)}\cos(x)\sin^2(x)\,dx$

$= \int \sin^{\frac{1}{2}}(x)\cos(x)\,dx - \int \sin^{\frac{1}{2}}(x)\cdot\sin^2(x)\cos(x)\,dx$

$$= \int \underbrace{\sin^{\frac{1}{2}}(x)}_{u^{1/2}} \underbrace{\cos(x)}_{u'} dx - \int \underbrace{\sin^{\frac{5}{2}}(x)}_{u^{5/2}} \underbrace{\cos(x)}_{u'} dx$$

$$= \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x) + C = \frac{2}{3}\sqrt{\sin^3(x)} - \frac{2}{7}\sqrt{\sin^7(x)} + C$$

$$= \boxed{\frac{2}{3}\sin(x)\sqrt{\sin(x)} - \frac{1}{7}\sin^3(x)\sqrt{\sin(x)} + C}$$

c) $\int \sin(5x)\cos(3x)\,dx$

Relations trigonométriques :

1) $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\Rightarrow \sin(5x+3x) = \sin(5x)\cos(3x) + \cos(5x)\sin(3x)$ (*)

2) $\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

$\Rightarrow \sin(5x-3x) = \sin(5x)\cos(3x) - \cos(5x)\sin(3x)$ (**)

$\Rightarrow$ (*) + (**) $\Rightarrow \sin(8x) + \sin(2x) = 2\sin(5x)\cos(3x)$

$\Rightarrow \sin(5x)\cos(3x) = \dfrac{\sin(8x) + \sin(2x)}{2}$

D'où $\int \sin(5x)\cos(3x)\,dx = \frac{1}{2}\int \left(\sin(8x) + \sin(2x)\right) dx$

$$= \frac{1}{2}\int \sin(8x)\,dx + \frac{1}{2}\int \sin(2x)\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{8}\int \underbrace{\sin(8x)}_{\sin(u)} \cdot \underbrace{8}_{u'} dx + \frac{1}{2}\cdot\frac{1}{2}\int \underbrace{\sin(2x)}_{\sin(u)} \cdot \underbrace{2}_{u'} dx$$

$$= \boxed{-\frac{1}{16}\cos(8x) - \frac{1}{4}\cos(2x) + C}$$

d) $\int \frac{\cos(x)}{2-\sin(x)}\,dx = (-1)\int \frac{(-1)\cos(x)}{2-\sin(x)}\,dx$ (u' au numérateur, u au dénominateur)

$= \boxed{-\ln(2-\sin(x)) + C}$

1.3.6 ~~2.2.8~~ Calculer :

a) $\int (3x^2 - 2x + 3)\,dx$

b) $\int \frac{3x^4 - 3x^2 - 7}{4x^2}\,dx$

c) $\int 7\sqrt[4]{x^3}\,dx$

d) $\int (\sqrt{x} - \sqrt[3]{x})\,dx$

e) $\int (2\sin(x) - 3\cos(x))\,dx$

f) $\int \cos(2x)\,dx$

e) $\int (2\sin(x) - 3\cos(x))\,dx = 2\int \sin(x)\,dx - 3\int \cos(x)\,dx$

$= \boxed{-2\cos(x) - 3\sin(x) + C}$

f) $\int \cos(2x)\,dx = \frac{1}{2}\int \cos(2x)\cdot 2\,dx = \boxed{\frac{1}{2}\sin(2x) + C}$ ($\cos(u)$ et u')

g) $\int \left(\frac{5}{\cos^2(x)} + 5\cos(x)\right)dx$

h) $\int \left(8\sin(x) + \frac{4}{\sqrt{2x}}\right)dx$

i) $\int (3x^2 - 7)^2\,dx$

j) $\int \sqrt{x}(x^2 - 5)\,dx$

k) $\int (3x - 5)^6\,dx$

l) $\int \frac{12}{(4-3x)^4}\,dx$

m) $\int \sqrt[3]{(3x-8)^2}\,dx$

n) $\int \frac{6}{\cos^2(3x)}\,dx$

o) $\int x\sqrt{x^2+1}\,dx$

p) $\int \frac{2x-1}{\sqrt{x^2-x-1}}\,dx$

g) $\int \left(\frac{5}{\cos^2(x)} + 5\cos(x)\right)dx = 5\int \frac{1}{\cos^2(x)}\,dx + 5\int \cos(x)\,dx$

$$= \boxed{5\tan(x) + 5\sin(x) + C}$$

$$\left((\tan(x))' = 1 + \tan^2(x) = \frac{1}{\cos^2(x)} \right)$$

h) $$\int \left(8\sin(x) + \frac{4}{\sqrt{2x}} \right) dx = 8\int \sin(x)\,dx + \int \frac{4}{\sqrt{2x}}\,dx$$

$$= 8\int \sin(x)\,dx + \int 2 \cdot \frac{2}{\sqrt{2x}}\,dx = \boxed{-8\cos(x) + 4\sqrt{2x} + C}$$

($u' = 2$, $\sqrt{u}$)

k) $$\int (3x-5)^6\,dx = \frac{1}{3}\int (3x-5)^6 \cdot 3\,dx = \frac{1}{3}\cdot\frac{1}{7}(3x-5)^7 + C$$

(u^6, u')

$$= \boxed{\frac{1}{21}(3x-5)^7 + C}$$

l) $$\int \frac{12}{(4-3x)^4}\,dx = \int 4 \cdot \frac{3}{(4-3x)^4}\,dx = 4(-1)\int (-1)\frac{3}{(4-3x)^4}\,dx$$

$$= -4\int \underbrace{(-3)}_{u'} \underbrace{(4-3x)^{-4}}_{u^{-4}}\,dx = -4 \cdot \frac{(4-3x)^{-3}}{(-3)} + C$$

$$= \frac{4}{3}(4-3x)^{-3} + C = \boxed{\frac{4}{3(4-3x)^3} + C}$$

m) $\int \sqrt[3]{(3x-8)^2}\,dx = \int (3x-8)^{2/3}\,dx = \frac{1}{3}\int \underbrace{(3x-8)^{2/3}}_{u^{2/3}}\,\underbrace{3}_{u'}\,dx$

$= \frac{1}{3}\,\frac{(3x-8)^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C = \frac{1}{3}\,\frac{(3x-8)^{\frac{5}{3}}}{\frac{5}{3}} + C$

$= \frac{1}{3}\cdot\frac{3}{5}\,(3x-8)^{\frac{5}{3}} + C = \boxed{\frac{1}{5}\sqrt[3]{(3x-8)^5} + C}$

n) $\int \frac{6}{\cos^2(3x)}\,dx = \int 2\cdot\frac{3}{\cos^2(3x)}\,dx = 2\int \underbrace{\frac{1}{\cos^2(3x)}}_{(\tan(u))'}\cdot\underbrace{3}_{u'}\,dx$

$= \boxed{2\tan(3x) + C}$

o) $\int x\sqrt{x^2+1}\,dx = \int x\,(x^2+1)^{1/2}\,dx = \frac{1}{2}\int \underbrace{(x^2+1)^{1/2}}_{u^{1/2}}\,\underbrace{2x}_{u'}\,dx$

$= \frac{1}{2}\cdot\frac{2}{3}\,(x^2+1)^{\frac{3}{2}} + C = \boxed{\frac{1}{3}\sqrt{(x^2+1)^3} + C}$

p) $\int \frac{\overbrace{2x-1}^{u'}}{\underbrace{\sqrt{x^2-x-1}}_{\sqrt{u}}}\,dx = \boxed{2\sqrt{x^2-x-1} + C}$

1.3.7 ~~2.2.9~~ Trouver l'expression mathématique de la fonction f, sachant que :

a) $f'(x) = 3x^2 - 4$, $f(5) = 54$;

b) $f''(x) = (x+1)(x-2)$, $f(1) = 8$, $f'(0) = 37/6$;

c) $f''(x) = \dfrac{1}{\sqrt{x}}$, $f'(9) = 2$, $f(1) = 2f(4)$.

a) $* \quad f'(x) = 3x^2 - 4 \Rightarrow f(x) = \int (3x^2 - 4)\,dx = \int 3x^2\,dx - \int 4\,dx$

$\Rightarrow f(x) = 3\,\frac{x^3}{3} - 4x + C$

$* \quad f(5) = 54 \Rightarrow f(5) = 3\cdot\frac{(5)^3}{3} - 4\cdot 5 + C = 54$

$\Rightarrow 125 - 20 + C = 54 \Rightarrow C = 54 - 125 + 20 = -51$

$\Rightarrow$ donc $\boxed{f(x) = x^3 - 4x - 51}$

b) $* \quad f''(x) = (x+1)(x-2) \Rightarrow f'(x) = \int f''(x)\,dx = \int (x+1)(x-2)\,dx$

$\Rightarrow f'(x) = \int (x^2 - 2x + x - 2)\,dx = \int (x^2 - x - 2)\,dx = \frac{x^3}{3} - \frac{x^2}{2} - 2x + C$

$* \quad f'(0) = \frac{37}{6} \Rightarrow \frac{0^3}{3} - \frac{0^2}{2} - 2\cdot 0 + C = \frac{37}{6} \Rightarrow C = \frac{37}{6}$

donc $f'(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{37}{6}$

$* f(x) = \int f'(x)\,dx = \int \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{37}{6}\right)dx = \frac{x^4}{3\cdot 4} - \frac{x^3}{2\cdot 3} - 2\frac{x^2}{2} + \frac{37}{6}x + C$

$* \quad f(1) = 8 \Rightarrow \frac{1}{12} - \frac{1}{6} - \frac{2}{2} + \frac{37}{6} + C = 8$

$\Rightarrow \frac{1}{12} - \frac{1}{6} - 1 + \frac{37}{6} + C = 8 \Rightarrow C = 8 - \frac{1}{12} + \frac{1}{6} + 1 - \frac{37}{6}$

$\Rightarrow C = \frac{96 - 1 + 2 + 12 - 74}{12} = \frac{35}{12}$

d'où $\boxed{f(x) = \frac{x^4}{12} - \frac{x^3}{6} - x^2 + \frac{37}{6}x + \frac{35}{12}}$

c) $* f'(x) = \int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 2x^{\frac{1}{2}} + C$

$* f'(9) = 2 \Rightarrow 2\cdot\sqrt{9} + C = 2 \Rightarrow 2\cdot 3 + C = 2$

$\Rightarrow C = -4 \Rightarrow f'(x) = 2\sqrt{x} - 4$

$* f(x) = \int (2\sqrt{x} - 4)\,dx = 2\int (\sqrt{x} - 2)\,dx = 2\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 4x + C$

$\Rightarrow f(x) = 2\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 4x + C = \frac{4}{3}\sqrt{x^3} - 4x + C$

$* f(1) = 2f(4)$

$\Rightarrow f(1) = \frac{4}{3}\sqrt{1} - 4 + C$ et $f(4) = \frac{4}{3}\sqrt{64} - 4\cdot 4 + C$

$= \frac{32}{3} - 16 + C$

$\Rightarrow \frac{4}{3} - 4 + C = 2\left(\frac{32}{3} - 16 + C\right) = \frac{64}{3} - 32 + 2C$

$\Rightarrow -C = \frac{64}{3} - 32 - \frac{4}{3} + 4 = \frac{64 - 96 - 4 + 12}{3} = -\frac{24}{3} = -8$

$\Rightarrow C = 8$

d'où $\boxed{f(x) = \frac{4}{3}\sqrt{x^3} - 2x + 8}$

1.3.9

2.2.11 Déterminer la fonction f sachant qu'elle admet pour asymptote la droite

$$x - 2y + 8 = 0$$

et que

$$f''(x) = -\frac{8}{x^3}$$

* $f'(x) = \int f''(x)\,dx = \int -\frac{8}{x^3}\,dx = -8\int x^{-3}\,dx = -8\cdot\frac{x^{-2}}{-2} + C$

$\Rightarrow f'(x) = \frac{4}{x^2} + C$

* $f(x) = \int f'(x)\,dx = \int \left(\frac{4}{x^2} + C\right)dx = 4\int (x^{-2} + C)\,dx$

$= 4\frac{x^{-1}}{-1} + Cx + d = -\frac{4}{x} + Cx + d$

* Asymptote : $x - 2y + 8 = 0 \Rightarrow y = \frac{x}{2} + 4$

Rappel : Théorème général de l'asymptote oblique :

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} \quad \text{et} \quad h = \lim_{x\to+\infty} (f(x) - mx)$$

$\Rightarrow$ d'où :

* $m = \lim_{x\to+\infty} \frac{-\frac{4}{x} + Cx + d}{x} = \lim_{x\to+\infty} -\frac{4}{x} + C + \frac{d}{x} = C$

$\Rightarrow C = \frac{1}{2}$

* $h = \lim_{x\to+\infty} \left(-\frac{4}{x} + \frac{1}{2}x + d\right) = d \Rightarrow d = 4$

Donc $f(x) = -\frac{4}{x} + \frac{1}{2}x + 4 \Rightarrow \boxed{f(x) = \frac{x^2 + 8x - 8}{2x}}$

1.3.10

~~2.2.12~~ Calculer :

a) $\int_1^4 (x^2 - 2x + 3)\,dx$

b) $\int_{-1}^1 (2x^3 + 3x^2 + 2x - 1)\,dx$

c) $\int_{-1}^1 \sqrt[3]{x+1}\,dx$

d) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin(x)\,dx$

e) $\int_0^{\frac{\pi}{4}} (1 + \tan^2(x))\,dx$

f) $\int_0^2 (1+2x)^3\,dx$

g) $\int_1^4 \frac{dx}{\sqrt{x}}$

h) $\int_0^{\frac{\pi}{2}} \sin^2(x)\cos(x)\,dx$

a) $\int_1^4 (x^2 - 2x + 3)\,dx = \left(\frac{x^3}{3} - \frac{2x}{2} + 3x\right)\Big|_1^4 = \frac{64}{3} - 4 - \frac{1}{3} - 2 = \boxed{15}$

b) $\int_{-1}^1 (2x^3 + 3x^2 + 2x - 1)\,dx = \left(\frac{2x^4}{4} + \frac{3x^3}{3} + \frac{2x^2}{2} - x\right)\Big|_{-1}^1 = \frac{1}{2} + 1 - \frac{1}{2} - 1 = \boxed{0}$

c) $\int_{-1}^1 \sqrt[3]{x+1}\,dx = \int_{-1}^1 (x+1)^{\frac{1}{3}}\,dx = \frac{(x+1)^{\frac{1}{3}+1}}{\frac{1}{3}+1}\Big|_{-1}^1$

$= \frac{3}{4}(x+1)^{\frac{4}{3}}\Big|_{-1}^1 = \frac{3}{4}\sqrt[3]{(x+1)^4}\Big|_{-1}^1 = \frac{3}{4}\sqrt[3]{2^4} - \frac{3}{4}\sqrt[3]{0}$

$= \frac{3}{4}\sqrt[3]{2^3 \cdot 2} = \frac{3 \cdot 2}{4}\sqrt[3]{2} = \boxed{\frac{3}{2}\sqrt[3]{2}}$

d) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin(x)\,dx = (-1)\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin(x)(-1)\,dx = -\cos(x)\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$= -\cos\left(\frac{3\pi}{4}\right) - \left(-\cos\left(\frac{\pi}{4}\right)\right) = -\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$

e) $\int_0^{\pi/4} \underbrace{(1+\tan^2(x))}_{\tan(x)'} dx = \tan(x)\Big|_0^{\pi/4} = \tan\left(\frac{\pi}{4}\right) - \tan(0) =$

$= 1 - 0 = \boxed{1}$

f) $\int_0^2 (1+2x)^3 dx \qquad \frac{1}{2}\int_0^2 \underbrace{(1+2x)^3}_{u^3}\underbrace{2}_{u'} dx = \frac{1}{2}\frac{(1+2x)^4}{4}\Big|_0^2$

$= \frac{1}{8}(1+2x)^4\Big|_0^2 = \frac{1}{8}(1+4)^4 - \frac{1}{8}\cdot 1 = \frac{1}{8}\cdot 625 - \frac{1}{8} = \boxed{78}$

g) $\int_1^4 \frac{dx}{\sqrt{x}} = \int_1^4 x^{-\frac{1}{2}} dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Big|_1^4 = 2x^{\frac{1}{2}}\Big|_1^4$

$= 2\sqrt{x}\Big|_1^4 = 2\sqrt{4} - 2\sqrt{1} = 2\cdot 2 - 2 = \boxed{2}$

h) $\int_0^{\pi/2} \underbrace{\sin^2(x)}_{u^2}\underbrace{\cos(x)}_{u'} dx = \int_0^{\pi/2} \sin^2(x)\cos(x)\,dx = \frac{\sin^3(x)}{3}\Big|_0^{\pi/2}$

$= \frac{1}{3}\sin^3\left(\frac{\pi}{2}\right) - \frac{1}{3}\sin^3(0) = \frac{1}{3}(1)^3 - \frac{1}{3}\cdot 0 = \boxed{\frac{1}{3}}$

1.3.11

2.2.13 Sachant que $\int_0^1 f(x)\,dx = 3$, $\int_1^2 f(x)\,dx = 4$ et $\int_2^3 f(x)\,dx = -8$, calculer :

a) $\int_0^2 f(x)\,dx$

b) $\int_0^1 3f(x)\,dx$

c) $\int_0^3 8f(x)\,dx$

d) $\int_3^1 2f(x)\,dx$

a) $\int_0^2 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx = 3 + 4 = \boxed{7}$

b) $\int_0^1 3f(x)\,dx = 3\int_0^1 f(x)\,dx = 3 \cdot 3 = \boxed{9}$

c) $\int_0^3 8f(x)\,dx = 8\int_0^3 f(x)\,dx = 8\left(\int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx\right) = 8\left(7 + (-8)\right) = \boxed{-8}$

d) $\int_3^1 2f(x)\,dx = -\int_1^3 2f(x)\,dx = -2\int_1^3 f(x)\,dx$

$= -2\left(\int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx\right) = -2\left(4 + (-8)\right) = \boxed{8}$

1.3.12 (MRS)

2.2.14 Montrer que pour une fonction f continue sur $[-a;a]$, on a :

a) $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ lorsque f est paire ;

b) $\int_{-a}^{a} f(x)\,dx = 0$ lorsque f est impaire.

a) $\int_{-a}^{0} f(t)\,dt = -\int_{0}^{-a} f(t)\,dt = -\int_{0}^{a} f(-x)\,(-1)\,dx = \int_{0}^{a} f(x)\,dx$

poser $t = -x$; $f(-x) = f(x)$; lucleur en t

$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ (cqfd)

$= \int_{0}^{a} f(x)\,dx$

b) $\int_{-a}^{0} f(t)\,dt = -\int_{0}^{-a} f(t)\,dt = -\int_{0}^{a} f(-x)\,(-1)\,dx = -\int_{0}^{a} f(x)\,dx$

poser $t = -x$; $f(-x) = -f(x)$; lucleur en t

$\Rightarrow \int_{-a}^{a} f(x) = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 0$ (cqfd)

$= -\int_{0}^{a} f(x)\,dx$

1.3.13

2.2.15 Déterminer les réels k pour lesquels on a :

a) $\displaystyle\int_{-1}^{2} kx^2\,dx = \frac{2}{3}$

b) $\displaystyle\int_{4}^{k} (x^2 - 3x + 7)\,dx = \frac{129}{2}$

c) $\displaystyle\int_{0}^{k/2} \cos(t)\,dt = \frac{1}{2}$

d) $\displaystyle\int_{k}^{0} \frac{2}{(x+1)^3}\,dx = -\int_{0}^{k} \frac{3}{(x+3)^2}\,dx$

a) $\displaystyle\int_{-1}^{2} kx^2\,dx = \frac{2}{3} \Rightarrow \left.\frac{kx^3}{3}\right|_{-1}^{2} = \frac{2}{3} \Rightarrow k\cdot\frac{2^3}{3} - k\frac{(-1)^3}{3} = \frac{2}{3}$

$\Rightarrow \frac{8}{3}k + \frac{k}{3} = \frac{2}{3} \Rightarrow 3k = \frac{2}{3} \Rightarrow \boxed{k = \frac{2}{9}}$

b) $\displaystyle\int_{4}^{k} (x^2 - 3x + 7)\,dx = \frac{129}{2} \Rightarrow \left(\frac{x^3}{3} - \frac{3x^2}{2} + 7x\right)\Big|_{4}^{k} = \frac{129}{2}$

$\Rightarrow \frac{k^3}{3} - \frac{3k^2}{2} + 7k - \frac{64}{3} - 4 = \frac{129}{2} \Rightarrow 2k^3 - 9k^2 + 42k - 539 = 0$

	2	−9	42	−539
7		14	35	
	2	5	77	0

$\Rightarrow (k-7)(\underbrace{2k^2 + 5k + 77}_{\Delta<0}) = 0 \Rightarrow \boxed{k = 7}$

c) $\displaystyle\int_{0}^{k/2} \cos(t)\,dt = \frac{1}{2} \Rightarrow \sin(t)\Big|_{0}^{\frac{k}{2}} = \frac{1}{2} \Rightarrow \sin\left(\frac{k}{2}\right) - 0 = \frac{1}{2}$

$\Rightarrow \begin{cases} \frac{k_1}{2} = \frac{\pi}{6} + \ell\cdot 2\pi \\ \frac{k_2}{2} = \pi - \frac{\pi}{6} + \ell 2\pi \end{cases} \Rightarrow \begin{cases} k_1 = \frac{\pi}{3} + \ell 4\pi \\ k_2 = \frac{5\pi}{3} + \ell 4\pi \end{cases}$

$\Rightarrow \boxed{k \in \left\{\frac{\pi}{3} + \ell 4\pi \;\middle|\; \ell \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{3} + \ell 4\pi \;\middle|\; \ell \in \mathbb{Z}\right\}}$

d)
$$\int_{k}^{0} \frac{2}{(x+1)^3}\,dx = -\int_{0}^{k} \frac{3}{(x+3)^2}\,dx$$

$$\Leftrightarrow \int_{k}^{0} 2(x+1)^{-3}\,dx = -\int_{0}^{k} 3(x+3)^{-2}\,dx \Leftrightarrow \left.2\frac{(x+1)^{-2}}{-2}\right|_{k}^{0} = -3\left.\frac{(x+3)^{-1}}{-1}\right|_{0}^{k}$$

$$\Leftrightarrow \left.-\frac{1}{(x+1)^2}\right|_{k}^{0} = \left.+\frac{3}{x+3}\right|_{0}^{k} \Leftrightarrow -\frac{1}{1} + \frac{1}{(k+1)^2} = \frac{3}{k+3} - 1$$

$$\Leftrightarrow -1 + \frac{1}{(k+1)^2} = \frac{3}{k+3} - 1 \Leftrightarrow \frac{1}{(k+1)^2} = \frac{3}{k+3} \Leftrightarrow k+3 = 3(k+1)^2$$

$$\Leftrightarrow k+3 = 3(k^2+2k+1) = k+3 = 3k^2+6k+3$$

$$\Leftrightarrow 3k^2+k = 0 \Leftrightarrow k(3k+5) = 0$$

$$\Leftrightarrow \boxed{k \in \left\{-\frac{5}{3}\,;\,0\right\}}$$