1.5.8 Soit $w = 1 + i\sqrt{3}$.

a) Calculer w^9

b) Calculer $\overline{w}^{-10}$

$w = 1 + i\sqrt{3} \Rightarrow |z| = r = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\Rightarrow \begin{cases} \cos(\theta) = \frac{1}{2} \\ \sin(\theta) = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \theta = \frac{\pi}{3}$

a) $w^9 = \left(2\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)\right)^9 = 2^9\left(\cos\left(9.\frac{\pi}{3}\right) + i\sin\left(9.\frac{\pi}{3}\right)\right)$

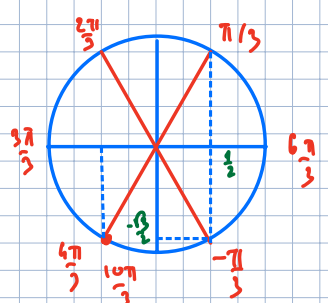
$= 2^9\left(\cos(3\pi) + i\sin(3\pi)\right) = 512\left(-1 + i.0\right)$

$\Rightarrow \boxed{w^9 = -512}$

b) $\overline{w}^{-10} \Rightarrow \overline{w} = 1 - i\sqrt{3}$

$|z| = r = \sqrt{1+3} = 2$

$\Rightarrow \begin{cases} \cos(\theta) = \frac{1}{2} \\ \sin(\theta) = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \theta = -\frac{\pi}{3}$



$\Rightarrow \overline{w}^{-10} = \left(2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)\right)^{-10}$

$= 2^{-10}\left(\cos\left((-10)\left(-\frac{\pi}{3}\right)\right) + i\sin\left((-10)\left(-\frac{\pi}{3}\right)\right)\right)$

$= 2^{-10}\left(\cos\left(\frac{10\pi}{3}\right) + i\sin\left(\frac{10\pi}{3}\right)\right) = 2^{-10}\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= \frac{1}{2^{10}}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{1}{1024} \cdot \frac{1}{2}\underbrace{\left(-1 - \sqrt{3}i\right)}_{r=2,\ \theta = -\frac{2\pi}{3}} = \frac{1}{2048}\left(-1 - \sqrt{3}i\right)$

$\Rightarrow \boxed{\overline{w}^{-10} = \frac{1}{1024} \cdot \frac{1}{2} 2.e^{-i\frac{2\pi}{3}} = \frac{1}{1024} e^{-i\frac{2\pi}{3}}}$