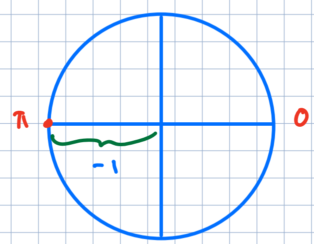
1.5.7 Calculer les racines cubiques de -8.

Soit $z = r(\cos(\theta) + i\sin(\theta))$ tel que $z^3 = r^3(\cos(3\theta) + i\sin(3\theta))$

et on sait que $z = -8 + 0i$

$$\Rightarrow \quad r = |z| = \sqrt{(-8)^2 + 0^2} = \sqrt{64} = 8$$

et $\begin{cases} \cos(\theta) = \dfrac{-8}{8} = -1 \\ \sin(\theta) = 0 \end{cases} \Rightarrow \theta = \pi$



$$\Rightarrow \quad 8(\cos(\pi) + i\sin(\pi)) = r^3(\cos(3\theta) + i\sin(3\theta))$$

$$\Rightarrow \begin{cases} 8 = r^3 \\ \pi + 2k\pi = 3\theta, \; k \in \mathbb{Z} \end{cases} \Rightarrow \begin{cases} r = 2 \\ \theta = \frac{\pi}{3} + k\frac{2\pi}{3}, \; k \in \mathbb{Z} \end{cases}$$

Donc • $z_1 = 2\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

ici, $k = 0 \Rightarrow \theta = \frac{\pi}{3}$

$$\Rightarrow \boxed{z_1 = 1 + \sqrt{3}\, i}$$

ici $k = 1$ • $z_2 = 2\left(\cos(\pi) + i\sin(\pi)\right) = 2(-1 + i \cdot 0)$

$\Rightarrow \theta = \frac{\pi}{3} + \frac{2\pi}{3} = \pi$

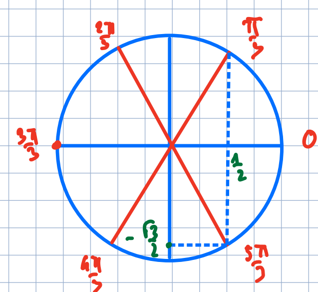
$$\Rightarrow \boxed{z_2 = -2}$$

ici $k = 2$ • $z_3 = 2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$

$\Rightarrow \theta = \frac{\pi}{3} + 2 \cdot \frac{2\pi}{3}$

$= \frac{5\pi}{3}$

$$= 2\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$$



$$\Rightarrow \boxed{z_3 = 1 - \sqrt{3}\, i}$$