1.5.2 Écrire sous forme polaire ($z = r \cdot e^{i\theta}$) les nombres complexes suivants :

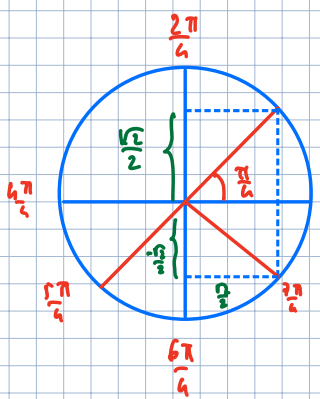
a) $z_1 = 1 - i$
b) $z_2 = -1 + i$
c) $z_3 = i$
d) $z_4 = 1 + i\sqrt{3}$
e) $z_5 = -\sqrt{3} - i$

Note : $z = r \cdot e^{i\theta} = r \cdot \text{cis}(\theta)$

Forme polaire : $z = r e^{i\theta}$

a) $z_1 = 1 - i \Rightarrow |z| = \sqrt{2}$

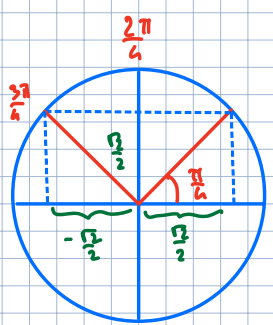
$$\begin{cases} \cos(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin(\theta) = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \end{cases} \Rightarrow \theta = \frac{7\pi}{4}$$



$$\Rightarrow \boxed{z_1 = \sqrt{2}\, e^{i\frac{7\pi}{4}}}$$

b) $z_2 = -1 + i$

$$\Rightarrow |z_2| = \sqrt{2} \Rightarrow \begin{cases} \cos(\theta) = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \\ \sin(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}$$



$$\Rightarrow \theta = \frac{3\pi}{4}$$

$$\Rightarrow \boxed{z_2 = \sqrt{2}\, e^{i\frac{3\pi}{4}}}$$

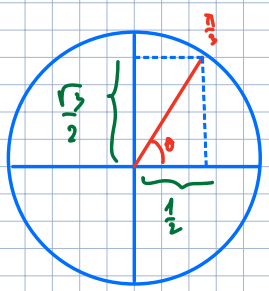
c) $z_3 = i$

$\Rightarrow |z_3| = 1 \quad \Rightarrow \begin{cases} \cos(\theta) = 1 \\ \sin(\theta) = 1 \end{cases} \Rightarrow \theta = \frac{\pi}{2}$

Donc $\boxed{z_3 = 1 \cdot e^{i\frac{\pi}{2}} = e^{i\frac{\pi}{2}}}$

d) $z_4 = 1 + i\sqrt{3}$

$\Rightarrow |z_4| = 2 \quad \Rightarrow \begin{cases} \cos(\theta) = \frac{1}{2} \\ \sin(\theta) = \frac{\sqrt{3}}{2} \end{cases}$

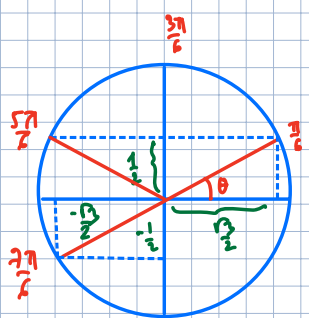


$\Rightarrow \theta = \frac{\pi}{3}$

Donc $\boxed{z_4 = 2\, e^{i\frac{\pi}{3}}}$

e) $z_5 = -\sqrt{3} - i$

$\Rightarrow |z_5| = 2 \quad \Rightarrow \begin{cases} \cos(\theta) = -\frac{\sqrt{3}}{2} \\ \sin(\theta) = -\frac{1}{2} \end{cases}$



$\Rightarrow \theta = \frac{7\pi}{6}$

Donc $\boxed{z_5 = 2\, e^{i\frac{7\pi}{6}}}$