Exercice 1.2.9

Déterminer sous forme trigonométrique et représenter dans le plan complexe :

a) les racines septièmes de l'unité

$$z^7 = 1 \qquad\qquad w = 1 \Rightarrow a = 1,\ b = 0$$

$$\Rightarrow z^7 = [1\,;0] \qquad\qquad |w| = 1$$

$$\begin{cases} \cos(\theta) = \dfrac{a}{r} = \dfrac{1}{1} = 1 \\ \sin(\theta) = \dfrac{b}{r} = 0 \end{cases}$$

$$\Rightarrow \theta = 0$$

On pose $z = [r\,;\theta]$

$$\Rightarrow z^7 = [r^7\,;\,7\theta] = [1\,;0]$$

$$\Rightarrow \begin{cases} r^7 = 1 \\ 7\theta = 2k\pi \end{cases} \Rightarrow \begin{cases} r = 1 \\ \theta = \dfrac{k}{7}\cdot 2\pi \end{cases} \quad k \in \mathbb{Z}$$

Donc $z_1 = [1\,;0]$, $z_2 = \left[1\,;\dfrac{2\pi}{7}\right]$, $z_3 = \left[1\,;\dfrac{4\pi}{7}\right]$

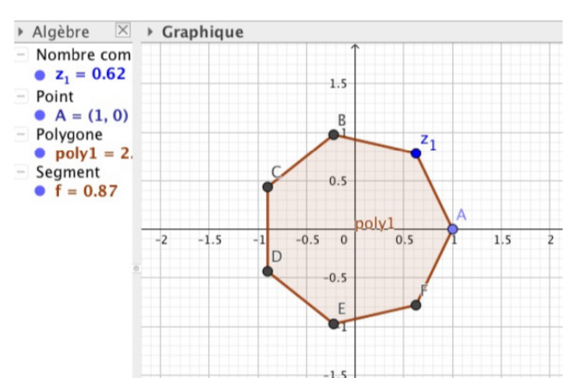
$z_4 = \left[1\,;\dfrac{6\pi}{7}\right]$, $z_5 = \left[1\,;\dfrac{8\pi}{7}\right]$, $z_6 = \left[1\,;\dfrac{10\pi}{7}\right]$

$z_7 = \left[1\,;\dfrac{12\pi}{7}\right]$

* représentation dans le plan complexe :

→ forme algébrique : $z = a + bi$

$\Rightarrow z_1 = [1, 0] = 1 + 0i$



$$\Rightarrow \quad z_2 = \left[1 ; \frac{2\pi}{7} \right] = 1 \left(\cos\left(\frac{2\pi}{7} \right) + i \sin\left(\frac{2\pi}{7} \right) \right.$$

$$z_2 = 0,62 + 0,78\, i$$

$$\Rightarrow \quad z_3 = \left[1 ; \frac{4\pi}{7} \right] = 1 \left(\cos\left(\frac{4\pi}{7} \right) + i \sin\left(\frac{4\pi}{7} \right) \right.$$

$$z_3 = -0,22 + 0,97\, i$$

$$\Rightarrow \quad z_4 = \left[1 ; \frac{6\pi}{7} \right] = 1 \left(\cos\left(\frac{6\pi}{7} \right) + i \sin\left(\frac{6\pi}{7} \right) \right)$$

$$z_4 = -0,90 + 0,43\, i$$

$$\Rightarrow \quad z_5 = \left[1 ; \frac{8\pi}{7} \right] = 1 \left(\cos\left(\frac{8\pi}{7} \right) + i \sin\left(\frac{8\pi}{7} \right) \right)$$

$$z_5 = -0,90 - 0,43\, i$$

$$z_6 = \left[1 ; \frac{10\pi}{7} \right] = 1 \left(\cos\left(\frac{10\pi}{7} \right) + i \sin\left(\frac{10\pi}{7} \right) \right)$$

$$z_6 = -0,22 - 0,97\, i$$

$$z_7 = \left[1 ; \frac{12\pi}{7} \right] = 1 \left(\cos\left(\frac{12\pi}{7} + i \sin\left(\frac{12\pi}{7} \right) \right) \right.$$

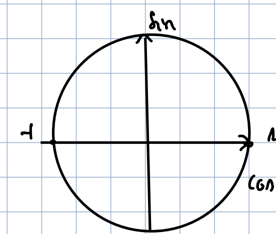
$$z_7 = 0,62 - 0,78\, i$$

5) les solutions de l'équation $z^5 = -32$

$$w = -32 = -32 + 0i$$

$$\Rightarrow a = -32 \ , \ b = 0 \quad \Rightarrow |w| = r = \sqrt{(-32)^2 + 0^2} = 32$$

$$\begin{cases} \cos(\theta) = \dfrac{a}{r} = \dfrac{-32}{32} = -1 \\ \sin(\theta) = \dfrac{b}{r} = \dfrac{0}{32} = 0 \end{cases} \Rightarrow \theta = \pi$$



$$\Rightarrow w = [32 ; \pi]$$

donc $z^5 = [32 ; \pi]$

On pose $z = [r ; \theta]$

$$\Rightarrow z^5 = [r^5 ; 5\theta]$$

d'où $[r^5 ; 5\theta] = [32 ; \pi]$

$$\Rightarrow \begin{cases} r^5 = 32 \\ 5\theta = \pi + 2k\pi , k \in \mathbb{Z} \end{cases} \Leftrightarrow \begin{cases} r^5 = 2^5 \\ \theta = \dfrac{\pi}{5} + \dfrac{2k\pi}{5} , k \in \mathbb{Z} \end{cases}$$

$$\Rightarrow \begin{cases} r = 2 \\ \theta = \dfrac{\pi}{5} + \dfrac{2k\pi}{5} , k \in \mathbb{Z} \end{cases}$$

Donc $z_1 = \left[2 ; \dfrac{\pi}{5}\right] = 2\left(\cos\left(\dfrac{\pi}{5}\right) + i \sin\left(\dfrac{\pi}{5}\right)\right)$

$$\Rightarrow z_1 = 2(0,81 + i\, 0,59)$$

$$\Rightarrow z_1 = 1,62 + 1,18i$$

$\Rightarrow z_2 = \left[2 \, ; \, \frac{3\pi}{5} \right] = 2 \left(\cos\left(\frac{3\pi}{5}\right) + i \sin\left(\frac{3\pi}{5}\right) \right)$

$z_2 = 2 \left(-0{,}31 + 0{,}95 i \right)$

$\Rightarrow z_2 = -0{,}62 + 1{,}9 i$

$\Rightarrow z_3 = \left[2 \, ; \, \pi \right] = 2 \left(\cos(\pi) + i \sin(\pi) \right)$

$z_3 = 2 \left(-1 + i \cdot 0 \right)$

$\Rightarrow z_3 = -2 + 0 \cdot i$

$\Rightarrow z_4 = \left[2 \, ; \, \frac{7\pi}{5} \right] = 2 \left(\cos\left(\frac{7\pi}{5}\right) + i \sin\left(\frac{7\pi}{5}\right) \right)$

$z_4 = 2 \left(-0{,}31 - 0{,}95 i \right)$

$\Rightarrow z_4 = -0{,}62 - 1{,}9 i$

$\Rightarrow z_5 = \left[2 \, ; \, \frac{9\pi}{5} \right] = 2 \left(\cos\left(\frac{9\pi}{5}\right) + i \sin\left(\frac{9\pi}{5}\right) \right)$

$z_5 = 2 \left(0{,}81 - 0{,}59 i \right)$

$\Rightarrow z_5 = 1{,}62 - 1{,}18 i$