**CORRIGÉ**

# Nombres complexes

## 1.1 L'ensemble des nombres complexes

**1.1.1** Exprimer les nombres complexes suivants sous la forme $a + b\,i$ :

a) $(1 + 4i) + (2 - 3i)$

b) $(8 + 5i) + (-8 - 5i)$

c) $(1 + i) - (2 - 6i)$

d) $(3 - 5i) + (-2 - 4i) - (1 - 2i)$

e) $3(5 - 2i) + 2(7 - i) - 3(4 - 3i)$

f) $(9 + 5i)(2 - 7i)$

g) $(3 + 2i)(3 - 2i)$

h) $(3 - 4i)^2$

i) $(1 + i)^4$

j) $\dfrac{1}{i}$

k) $\dfrac{1}{2 + 3i}$

l) $\dfrac{1 + i}{1 - i}$

m) $\dfrac{5 + 3i}{2 + 4i}$

n) $\left(\dfrac{63 + 16i}{4 + 3i}\right)^2$

a) $(1 + 4i) + (2 - 3i) = 1 + 4i + 2 - 3i = \boxed{3 + i}$

b) $(8 + 5i) + (-8 - 5i) = 8 + 5i - 8 - 5i = \boxed{0}$

c) $(1 + i) - (2 - 6i) = 1 + i - 2 + 6i = \boxed{-1 + 7i}$

d) $(3 - 5i) + (-2 - 4i) - (1 - 2i) = 3 - 5i - 2 - 4i - 1 + 2i = 0 - 7i = \boxed{-7i}$

e) $3(5 - 2i) + 2(7 - i) - 3(4 - 3i) = 15 - 6i + 14 - 2i - 12 + 9i \boxed{= 17 + i}$

f) $(9 + 5i)(2 - 7i) = 18 - 63i + 10i - 35i^2 = 18 - 35(-1) - 63i + 10i \boxed{= 53 - 53i}$

$i^2 = -1$

g) $(3 + 2i)(3 - 2i) = 9 - 4i^2 = 9 + 4 \boxed{= 13}$

h) $(3 - 4i)^2 = 9 - 24i + 16i^2 = 9 - 24i - 16 = \boxed{-7 - 24i}$

i) $(1+i)^4 = ((1+i)^2)^2 = (1+2i+i^2)^2 = (1+2i-1)^2 = 4i^2 \boxed{=-4}$

j) $\dfrac{1}{i} = \dfrac{1}{i} \cdot \dfrac{i}{i} = \dfrac{i}{i^2} = \boxed{-i}$

k) $\dfrac{1}{2+3i} = \dfrac{1}{2+3i} \cdot \dfrac{2-3i}{2-3i} = \dfrac{2-3i}{(2+3i)(2-3i)} = \dfrac{2-3i}{4-\underbrace{9i^2}_{=-1}}$

$= \dfrac{2-3i}{4+9} = \boxed{\dfrac{2}{13} - \dfrac{3}{13} i}$

l) $\dfrac{1+i}{1-i} = \dfrac{(1+i)(1+i)}{(1-i)(1+i)} = \dfrac{(1+i)^2}{1-i^2} = \dfrac{1+2i+\overbrace{i^2}^{=-1}}{2} = \dfrac{2i}{2} \boxed{= i}$

m) $\dfrac{5+3i}{2+4i} = \dfrac{(5+3i)(2-4i)}{(2+4i)(2-4i)} = \dfrac{10-20i+6i-12\overbrace{i^2}^{=-1}}{4-16\underbrace{i^2}_{=-1}} = \dfrac{22-14i}{20}$

$= \dfrac{22}{20} - \dfrac{14}{20} i = \boxed{\dfrac{11}{10} - \dfrac{7}{10} i}$

n) $\left(\dfrac{63+16i}{4+3i}\right)^2 = \dfrac{3969+2016i+256i^2}{16+24i+9i^2} = \dfrac{3713+2016i}{7+24i}$

$= \dfrac{(3713+2016i)(7-24i)}{(7+24i)(7-24i)} = \dfrac{25991 - 89112i + 14112i - \overbrace{48384i^2}^{=-48384}}{49 - \underbrace{576i^2}_{=-576}}$

$= \dfrac{74375 - 75000i}{625} = \dfrac{74375}{625} - \dfrac{75000}{625} i$

$= \boxed{119 - 120i}$

méthode conseillée !

ou $\left(\dfrac{63+16i}{4+3i} \cdot \dfrac{4-3i}{4-3i}\right)^2 = \left(\dfrac{252-189i+64i+48}{16+9}\right)^2 = (12-5i)^2 \boxed{= 119-120i}$

**1.1.3** Résoudre dans $\mathbb{C}$ les équations ci-dessous :

a) $2z - 3 + i = 0$

b) $(1-4i)z = 6 - 7i$

c) $(1+2i)z = (5-i)z + 7 + 26i$

a) $2z - 3 + i = 0 \Rightarrow 2z = 3 - i$

$\Rightarrow z = \frac{3}{2} - \frac{i}{2} \Rightarrow \boxed{S = \left\{ \frac{3}{2} - \frac{i}{2} \right\}}$

b) $(1-4i)z = 6 - 7i$

$\Rightarrow z = \frac{-6+7i}{-1+4i}$

$\Rightarrow z = \frac{(-6+7i)(-1-4i)}{(-1+4i)(-1-4i)} = \frac{6+24i-7i-28i^2}{1-16i^2} = \frac{34+17i}{17}$

$\Rightarrow z = \frac{34}{17} + \frac{17}{17}i = 2 + i \Rightarrow \boxed{S = \{2+i\}}$

c) $(1+2i)z = (5-i)z + 7 + 26i$

$\Rightarrow (1+2i)z - (5-i)z = 7 + 26i$

$\Rightarrow z(1+2i-5+i) = 7 + 26i$

$\Rightarrow z(-4+3i) = 7 + 26i \Rightarrow z = \frac{7+26i}{-4+3i}$

$\Rightarrow z = \frac{7+26i}{-4+3i} \cdot \frac{-4-3i}{-4-3i} = \frac{7+26i}{-4+3i} \cdot \frac{4+3i}{4+3i} = \frac{28+21i+104i-78}{-25}$

$= \frac{-50+125i}{-25} = 2 - 5i \Rightarrow \boxed{S = \{2-5i\}}$

**1.1.4** Résoudre dans $\mathbb{C}$ le système d'équations ci-dessous :

$$\begin{cases} (2+i)z + (2-i)w &= 7-4i \\ (1+i)z - iw &= 2+i \end{cases} \quad (*)$$

$$\begin{cases} (2+i)z + (2-i)w = 7-4i & \cdot\, i \\ (1+i)z - iw = 2+i & \cdot\, (2-i) \end{cases}$$

$$\Leftrightarrow \begin{cases} i(2+i)z + i(2-i)w = (7-4i)\cdot i \\ \underbrace{(2-i)(1+i)}_{=2+2i-i-i^2=3+i} z - (2-i)iw = \underbrace{(2+i)(2-i)}_{=4-2i+2i-i^2=5} \end{cases}$$

$$+ \begin{cases} (2i-1)z + i(2-i)w = 4+7i \\ (3+i)z - i(2-i)w = 5 \end{cases}$$

$$\Rightarrow (2i-1)z + (3+i)z = 9+7i$$

$$\Rightarrow z(2i-1+3+i) = 9+7i$$

$$\Rightarrow z(3i+2) = 9+7i \Rightarrow z = \frac{9+7i}{2+3i}$$

$$\Rightarrow z = \frac{9+7i}{2+3i}\cdot\frac{2-3i}{2-3i} = \frac{39-13i}{4+9} = 3-i$$

$$(*) \Rightarrow (1+i)(3-i) - iw = 2+i \Rightarrow 3-i+3i+1-iw = 2+i$$

$$\Rightarrow iw = 2+i \Rightarrow w = \frac{2+i}{i} = \frac{2+i}{i}\,\frac{i}{i} = \frac{2i+i^2}{-1}$$

$$\Rightarrow w = \frac{-1+2i}{-1} = 1-2i$$

Donc $\boxed{S = \{(1-2i)\,;\,(3-i)\}}$

**1.1.5** Déterminer le conjugué des nombres complexes ci-dessous :

a) $z_1 = 5 - 4i$

b) $z_2 = -8 - i$

c) $z_3 = 2 + 3i$

d) $z_4 = 5 - \frac{3}{2}i$

a) $z_1 = 5 - 4i \Rightarrow \boxed{\overline{z_1} = 5 + 4i}$

b) $z_2 = -8 - i \Rightarrow \boxed{\overline{z_2} = -8 + i}$

c) $z_3 = 2 + 3i \Rightarrow \boxed{\overline{z_3} = 2 - 3i}$

d) $z_4 = 5 - \frac{3}{2}i \Rightarrow \boxed{\overline{z_4} = 5 + \frac{3}{2}i}$

**1.1.6** Calculer $(7 - 8i)\overline{(8 - 7i)} - \overline{(4 + 3i)}(4 - 2i)$.

$$(7-8i)\overline{(8-7i)} - \overline{(4+3i)}(4-2i) = (7-8i)(8+7i) - (4-3i)(4-2i)$$

$$= 56 + 49i - 64i + 56 - (16 - 8i - 12i - 6)$$

$$= \boxed{102 + 5i}$$

**1.1.7** Poser $z = x + yi$ et résoudre dans $\mathbb{C}$ les équations ci-dessous :

a) $8z + 5\bar{z} = 4 + 3i$

b) $z^2 + 2\bar{z} + 5 = 0$

c) $2\,\Im(\bar{z} + 1) + 2i\,\Re(-z + 2) = -1 - 12i$

a) $8z + 5\bar{z} = 4 + 3i$ $\qquad z = x + yi \Rightarrow \bar{z} = x - yi$

$\Rightarrow 8(x + yi) + 5(x - yi) = 4 + 3i \Rightarrow 13x + 3yi = 4 + 3i$

Donc $x = \frac{4}{13}$ et $y = 1$

$\Rightarrow \boxed{S = \left\{ \frac{4}{13} + i \right\}}$

b) $z^2 + 2\bar{z} + 5 = 0$

$\Rightarrow (x + yi)^2 + 2(x - yi) + 5 = 0$

$\Rightarrow x^2 + 2xyi + y^2 i^2 + 2x - 2yi + 5 = 0$

$\Rightarrow x^2 + 2xyi - y^2 + 2x - 2yi + 5 = 0$

$\Rightarrow x^2 - y^2 + 2x + i(2xy - 2y) = -5 = -5 + 0i$

$\Rightarrow \begin{cases} x^2 - y^2 + 2x = -5 \\ 2xy - 2y = 0 \end{cases} \Rightarrow 2y(x - 1) = 0$

• $y = 0 \Rightarrow x^2 + 2x = -5 \Rightarrow x^2 + 2x + 5 = 0 \Rightarrow \Delta = -16 < 0$
pas de solutions

• $x - 1 = 0 \Rightarrow x = 1 \Rightarrow 3 - y^2 = -5 \Rightarrow y^2 = 8 \Rightarrow y = \pm 2\sqrt{2}$

$\Rightarrow \boxed{S = \left\{ 1 \pm 2\sqrt{2}\, i \right\}}$

c) $$2\,\underbrace{\mathcal{I}}_{\text{partie imaginaire}}(\bar{z}+1) + 2i\,\underbrace{\mathcal{R}}_{\text{partie réelle}}(-z+2) = -1-12i$$

$$\Rightarrow \quad 2\mathcal{I}(x-yi+1) + 2i\,\mathcal{R}(-x-yi+2) = -1-12i$$

$$\Rightarrow \quad 2\mathcal{I}(x+1-yi) + 2i\,\mathcal{R}(-x+2-yi) = -1-12i$$

$$\Rightarrow \quad 2(-y) + 2i(-x+2) = -1-12i$$

$$\Rightarrow \quad -2y + (-2x+4)i = -1-12i$$

$$\Rightarrow \quad \begin{cases} -2y = -1 \\ -2x+4 = -12 \end{cases} \Rightarrow \begin{cases} y = \frac{1}{2} \\ -2x = -16 \end{cases} \Rightarrow \begin{cases} y = \frac{1}{2} \\ x = +8 \end{cases}$$

$$\Rightarrow \quad \boxed{S = \left\{8 + \frac{1}{2}i\right\}}$$

**1.1.8** Soit $z$ un nombre complexe. Démontrer que

$$z + \bar{z} = 2\,\Re(z), \quad z - \bar{z} = 2\,\Im(z)\,i \quad \text{et} \quad z\bar{z} = \Re(z)^2 + \Im(z)^2$$

Soient $z = a + bi$ et $\bar{z} = a - bi$

$\Rightarrow \quad z + \bar{z} = a + bi + a - bi = 2a = 2\,R(z)$ ✓

$\Rightarrow \quad z - \bar{z} = a + bi - a + bi = 2bi = 2\,R(z)\,i$ ✓

$\Rightarrow \quad z \cdot \bar{z} = (a+bi)(a-bi) = a^2 - b^2 i^2 = a^2 + b^2 = R(z)^2 + I(z)^2$ ✓

**1.1.9** Montrer que si $w$ est une solution de l'équation réelle $az^2 + bz + c = 0$, alors $\bar{w}$ en est une aussi.

Si $w$ est une solution de $az^2 + bz + c = 0$

$\Rightarrow \quad aw^2 + bw + c = 0$

$\Rightarrow \quad \overline{aw^2 + bw + c} = \bar{0}$

$\Rightarrow \quad \overline{aw^2} + \overline{bw} + \bar{c} = 0$

$\Rightarrow \quad a\overline{w^2} + b\bar{w} + c = 0$

$\Rightarrow \quad a\bar{w}^2 + b\bar{w} + c = 0$

Alors $\bar{w}$ est aussi une solution.

## 1.2 Nombres complexes sous forme trigonométrique

**1.2.1** Représenter les points $A$, $B$, ..., $H$ dans le plan complexe après avoir calculé, si nécessaire, leurs affixes $z_A$, $z_B$, ..., $z_H$ :

a) $z_A = 2 - i$

b) $z_B = -3 + 2i$

c) $z_C = z_A + z_B = 2 - i - 3 + 2i = -1 + i$

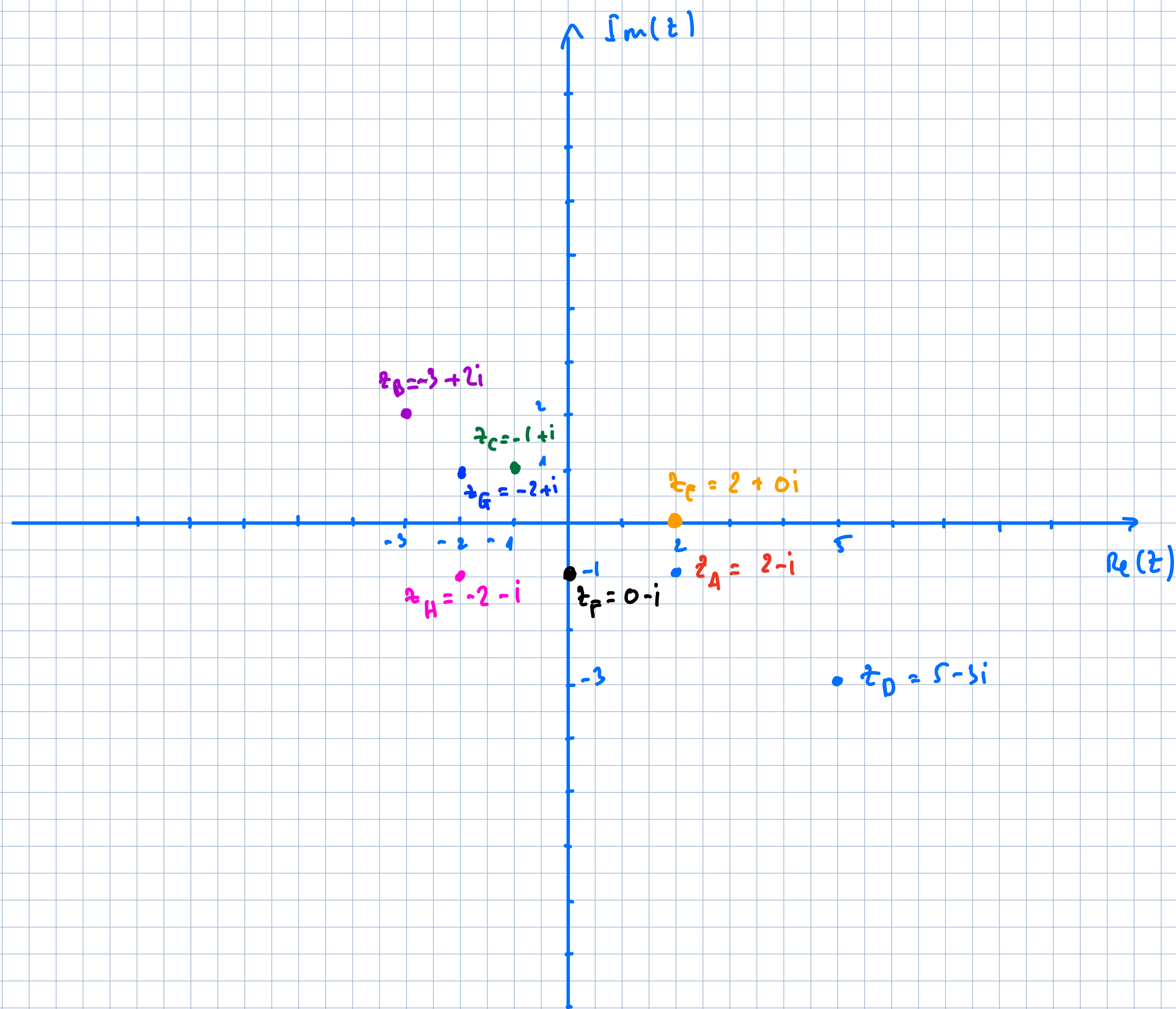
d) $z_D = z_A - z_B = 2 - i + 3 - 2i = 5 - 3i$

e) $z_E = \dfrac{z_A + \overline{z_A}}{2} = \dfrac{2 - i + 2 + i}{2} = 2 + 0i$

f) $z_F = \dfrac{z_A - \overline{z_A}}{2} = \dfrac{2 - i - (2 + i)}{2} = -i = 0 - i$

g) $z_G = -z_A = -2 + i$

h) $z_H = -\overline{z_A} = -(2 + i) = -2 - i$



**1.2.2** Écrire les nombres complexes ci-dessous sous forme trigonométrique :

a) $1$

b) $i$

c) $-2$

d) $-1-i$

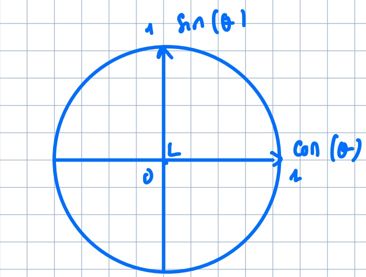
e) $-1-\sqrt{3}i$

f) $3+4i$

a) $z = 1 = 1 + 0i$

$\Rightarrow |z| = \sqrt{1^2 + 0^2} = 1 = r$

$$\begin{cases} \cos(\theta) = \frac{a}{r} = \frac{1}{1} = 1 \\ \sin(\theta) = \frac{b}{r} = \frac{0}{1} = 0 \end{cases}$$



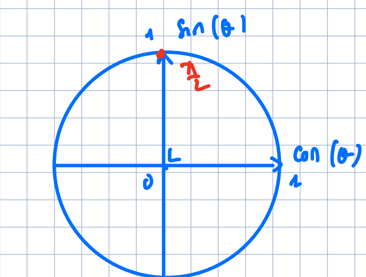

$\Rightarrow \theta = 0$

$\Rightarrow$ $\boxed{z = r \operatorname{cis}(\theta) = 1 \operatorname{cis}(0)}$

b) $z = i = 0 + i$

$\Rightarrow |z| = r = 1$

$$\begin{cases} \cos(\theta) = \frac{a}{r} = 0 \\ \sin(\theta) = \frac{b}{r} = \frac{1}{1} = 1 \end{cases}$$



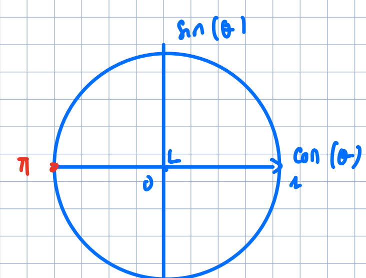

$\Rightarrow \theta = \frac{\pi}{2} + 2k\pi, \ k \in \mathbb{Z}$

$\Rightarrow$ $\boxed{z = 1 \operatorname{cis}\left(\frac{\pi}{2}\right)}$

c) $z = -2 = -2 + 0i$

$\Rightarrow |z| = 2$

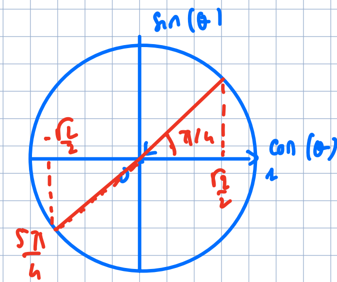
$$\begin{cases} \cos(\theta) = -1 \\ \sin(\theta) = 0 \end{cases} \Rightarrow \theta = \pi + 2k\pi, \ k \in \mathbb{Z}$$




$\Rightarrow$ $\boxed{z = 2 \operatorname{cis}(\pi)}$

d) $z = -1 - i$

$\Rightarrow |z| = r = \sqrt{2}$



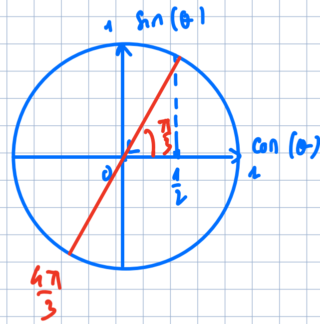

$$\Rightarrow \begin{cases} \cos(\theta) = \dfrac{-1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2} \\ \sin(\theta) = \dfrac{-1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2} \end{cases}$$

$\Rightarrow \theta = \dfrac{5\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$

$\rightarrow \boxed{z = \sqrt{2} \operatorname{cis}\left(\dfrac{5\pi}{4}\right)}$

e) $z = -1 - \sqrt{3}i$

$\Rightarrow |z| = r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$




$$\Rightarrow \begin{cases} \cos(\theta) = \dfrac{a}{r} = -\dfrac{1}{2} \\ \sin(\theta) = \dfrac{b}{r} = -\dfrac{\sqrt{3}}{2} \end{cases}$$

$\Rightarrow \theta = \dfrac{4\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$

$\Rightarrow \boxed{z = 2 \operatorname{cis}\left(\dfrac{4\pi}{3}\right)}$

f) $z = 3 + 4i$

$\Rightarrow |z| = r = \sqrt{9 + 16} = 5$

$$\Rightarrow \begin{cases} \cos(\theta) = \dfrac{3}{5} \\ \sin(\theta) = \dfrac{4}{5} \end{cases} \Rightarrow \theta \cong 53{,}13^\circ$$

$\Rightarrow \boxed{z = 5 \operatorname{cis}(53{,}13^\circ)}$

**1.2.3** Écrire les nombres complexes ci-dessous sous forme algébrique :

a) $\left[4; -\frac{\pi}{3}\right]$

b) $\left[\frac{3}{4}; \frac{3\pi}{4}\right]$

c) $[\pi; -\pi]$

d) $\left[4; \frac{\pi}{3}\right]$

e) $\left[1; -\frac{\pi}{2}\right]$

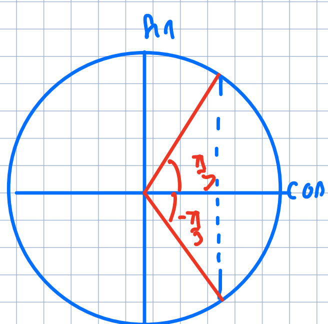
f) $e^{i\pi}$

Rappel : Forme algébrique $z = a + bi$

a) $\left[4 ; -\frac{\pi}{3}\right] \Rightarrow r = 4 \;, \; \theta = -\frac{\pi}{3}$

$\Rightarrow z = a + bi = r\cos\left(-\frac{\pi}{3}\right) + r\sin\left(-\frac{\pi}{3}\right)i$

On sait que $\begin{cases} \cos(\theta) = \frac{a}{r} \\ \sin(\theta) = \frac{b}{r} \end{cases} \Rightarrow \begin{cases} \cos\left(-\frac{\pi}{3}\right) = \frac{a}{4} \\ \sin\left(-\frac{\pi}{3}\right) = \frac{b}{4} \end{cases} \Rightarrow \begin{cases} a = 4\cos\left(-\frac{\pi}{3}\right) \\ b = 4\sin\left(-\frac{\pi}{3}\right) \end{cases}$




$\Rightarrow \begin{cases} a = 4\cos\left(\frac{\pi}{3}\right) = 4 \cdot \frac{1}{2} = 2 \\ b = -4\sin\left(\frac{\pi}{3}\right) = -4 \cdot \frac{\sqrt{3}}{2} \;\; -2\sqrt{3} \end{cases}$

$\Rightarrow \boxed{z = 2 - 2\sqrt{3}\, i}$

plus simple :

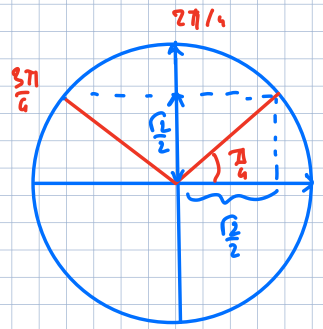
$z = a + bi = r\left(\cos(\theta) + i\sin(\theta)\right) = 4\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

$= 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 4 \cdot \frac{1}{2} - \frac{4\sqrt{3}}{2}i = \boxed{2 - 2\sqrt{3}\, i}$

b) $\left[ \frac{3}{4} ; \frac{3\pi}{4} \right]$

$$\Rightarrow z = \frac{3}{4}\left( \cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right) = \frac{3}{4}\left( -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right)$$

$$\Rightarrow \boxed{z = -\frac{3\sqrt{2}}{8} + \frac{3\sqrt{2}}{8} i}$$

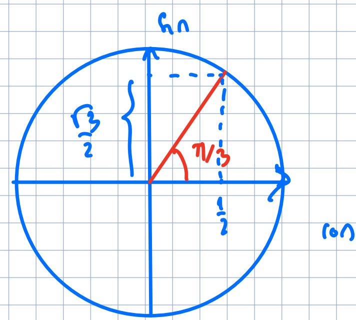



c) $[\pi ; -\pi]$

$$\Rightarrow z = \pi\left( \cos(-\pi) + i \sin(-\pi) \right) = \pi(-1 + 0i)$$

$$\boxed{z = -\pi}$$

d) $\left[ 4 ; \frac{\pi}{3} \right]$

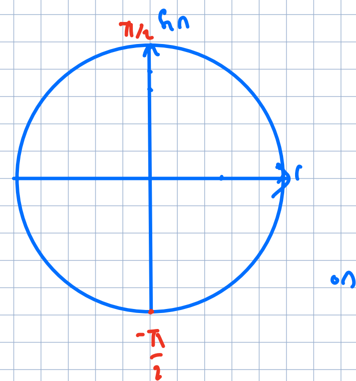



$$\Rightarrow z = 4\left( \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right)$$

$$= 4\left( \frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$$

$$\Rightarrow \boxed{z = 2 + 2\sqrt{3} i}$$

e) $\left[ 1 ; -\frac{\pi}{2} \right]$




$$\Rightarrow z = 1\left( \cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right) \right) =$$

$$= 1(0 - 1 \cdot i)$$

$$\Rightarrow \boxed{z = -i}$$

f) $e^{i\pi}$

$$\Rightarrow z = a + bi = re^{i\theta} \Rightarrow r = 1 \text{ et } \theta = \pi$$

$$\Rightarrow z = 1\left( \cos(\pi) + i \sin(\pi) \right) = 1(-1 + 0i) \Rightarrow \boxed{z = -1}$$

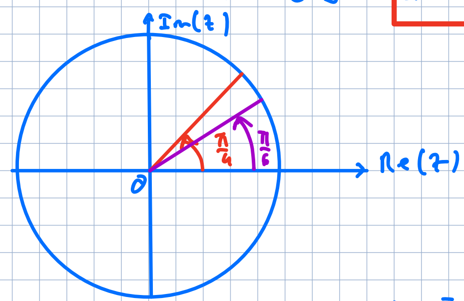
**1.2.4** Calculer :

a) $\left[2; \frac{\pi}{4}\right] \cdot \left[3; \frac{\pi}{6}\right]$ b) $\left[6; \frac{2\pi}{3}\right] : \left[3; -\frac{\pi}{3}\right]$ c) $\left[2; \frac{\pi}{3}\right]^3$

Rappel :

$$[r_1 ; \theta_1] \cdot [r_2 ; \theta_2] = [r_1 \cdot r_2 ; \theta_1 + \theta_2]$$

a) $$\underbrace{\left[2 ; \frac{\pi}{4}\right]}_{z_1} \cdot \underbrace{\left[3 ; \frac{\pi}{6}\right]}_{z_2} = \left[2 \cdot 3 ; \frac{\pi}{4} + \frac{\pi}{6}\right] = \boxed{\left[6 ; \frac{5\pi}{12}\right]}$$




b) $$\left[6 ; \frac{2\pi}{3}\right] : \left[3 ; -\frac{\pi}{3}\right] = \left[\frac{6}{3} ; \frac{2\pi}{3} - \left(-\frac{\pi}{3}\right)\right] = \left[2 ; \frac{2\pi}{3} + \frac{\pi}{3}\right]$$

$$= \boxed{[2 ; \pi]}$$

ou $$(6 : 3) \operatorname{cis}\left(\frac{2\pi}{3} + \frac{\pi}{3}\right) = (2) \operatorname{cis}(\pi) \Rightarrow \boxed{[2 ; \pi]}$$

c) $$\left[2 ; \frac{\pi}{3}\right]^3$$

Rappel : $z^n = r^n \operatorname{cis}(n\theta)$

$$\Rightarrow \left[2 ; \frac{\pi}{3}\right]^3 = 2^3 \operatorname{cis}\left(3 \cdot \frac{\pi}{3}\right) = 8 \operatorname{cis}(\pi)$$

$$\Rightarrow \boxed{[8 ; \pi]}$$

**1.2.5** Calculer $z_1 z_2$ et $z_1/z_2$ en utilisant la forme trigonométrique :

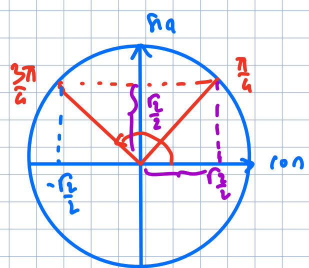
a) $z_1 = -1 + i, \quad z_2 = 1 + i$

b) $z_1 = -2 - 2\sqrt{3}\,i, \quad z_2 = 5\,i$

c) $z_1 = 2\,i, \quad z_2 = -3\,i$

d) $z_1 = -10, \quad z_2 = -4$

a) $z_1 = -1 + i \quad , \; z_2 = 1 + i$




* $|z_1| = \sqrt{1+1} = \sqrt{2}$

$$\Rightarrow \begin{cases} \cos(\theta_1) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin(\theta_1) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases} \Rightarrow \theta = \frac{3\pi}{4}$$

$$\Rightarrow z_1 = \sqrt{2}\,\mathrm{cis}\left(\frac{3\pi}{4}\right)$$

* $|z_2| = \sqrt{2}$

$$\begin{cases} \cos(\theta_2) = \frac{\sqrt{2}}{2} \\ \sin(\theta_2) = \frac{\sqrt{2}}{2} \end{cases} \Rightarrow \theta_2 = \frac{\pi}{4} \quad \Rightarrow z_2 = \sqrt{2}\,\mathrm{cis}\left(\frac{\pi}{4}\right)$$

i) $\Rightarrow z_1 \cdot z_2 = [\, r_1 \cdot r_2 \; ; \; \theta_1 + \theta_2 \,] = [\, \sqrt{2} \cdot \sqrt{2} \; ; \; \frac{3\pi}{4} + \frac{\pi}{4} \,]$

$z_1 \cdot z_2 = [\, 2 \; ; \; \pi \,] = 2\left(\cos(\pi) + i\sin(\pi)\right)$

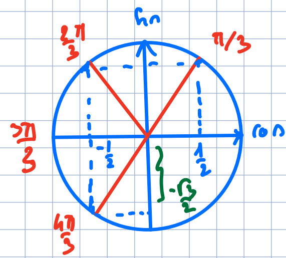
$\Rightarrow z_1 \cdot z_2 = 2(-1 + 0i) \Rightarrow \boxed{z_1 \cdot z_2 = -2}$

ii) $\Rightarrow \frac{z_1}{z_2} = \left[\frac{r_1}{r_2} \; ; \; \theta_1 - \theta_2\right] = \left[\frac{\sqrt{2}}{\sqrt{2}} \; ; \; \frac{3\pi}{4} - \frac{\pi}{4}\right] = \left[1 \; ; \; \frac{\pi}{2}\right]$

$\Rightarrow \frac{z_1}{z_2} = 1\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) = 1(0 + i)$

$\Rightarrow \boxed{\frac{z_1}{z_2} = i}$

b) $z_1 = -2 - 2\sqrt{3}\,i \quad , \quad z_2 = 5i$




$\Rightarrow |z_1| = r_1 = 4 \qquad \Rightarrow \begin{cases} \cos(\theta_1) = -\frac{1}{2} \\ \sin(\theta_1) = -\frac{\sqrt{3}}{2} \end{cases}$

$\Rightarrow \theta_1 = \frac{4\pi}{3} \quad \Rightarrow z_1 = 4\,\mathrm{cis}\left(\frac{4\pi}{3}\right)$

$\Rightarrow |z_2| = 5 \quad \Rightarrow \begin{cases} \cos(\theta_2) = 0 \\ \sin(\theta_2) = 1 \end{cases} \quad \Rightarrow \theta_2 = \frac{\pi}{2} \quad \Rightarrow z_2 = 5\,\mathrm{cis}\left(\frac{\pi}{2}\right)$

i) $z_1 \cdot z_2 = \left[4 \cdot 5 \ ; \ \frac{4\pi}{3} + \frac{\pi}{2}\right] = \left[20 \ ; \ \frac{8\pi + 3\pi}{6}\right] = \left[20 \ ; \ \frac{11\pi}{6}\right]$

$\Rightarrow z_1 \cdot z_2 = 20\,\mathrm{cis}\left(\frac{11\pi}{6}\right) = 20\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$

$= 20\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) \Rightarrow \boxed{z_1 \cdot z_2 = 10\sqrt{3} - 10i}$

ii) $\frac{z_1}{z_2} = \left[\frac{r_1}{r_2} \ ; \ \theta_1 - \theta_2\right] = \left[\frac{4}{5} \ ; \ \frac{4\pi}{3} - \frac{\pi}{2}\right] = \left[\frac{4}{5} \ ; \ \frac{5\pi}{6}\right]$

$= \frac{4}{5}\,\mathrm{cis}\left(\frac{5\pi}{6}\right)$

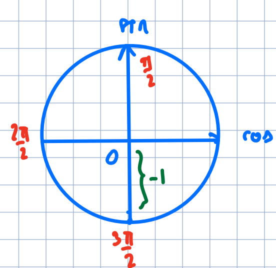
$= \frac{4}{5}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right) = \frac{4}{5}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$\Rightarrow \boxed{\frac{z_1}{z_2} = \frac{-2\sqrt{3}}{5} + \frac{2i}{5}}$

c) $z_1 = 2i \quad , \quad z_2 = -3i$

$\Rightarrow |z_1| = r_1 = 2 \quad \Rightarrow \begin{cases} \cos(\theta_1) = 0 \\ \sin(\theta_1) = 1 \end{cases} \quad \Rightarrow \theta_1 = \frac{\pi}{2}$

$\Rightarrow z_1 = 2\,\mathrm{cis}\left(\frac{\pi}{2}\right)$




$\Rightarrow |z_2| = r_2 = 3 \quad \Rightarrow \begin{cases} \cos(\theta_2) = 0 \\ \sin(\theta_2) = -1 \end{cases}$

$\Rightarrow \theta_2 = \frac{3\pi}{2}$

$\Rightarrow z_2 = 3\,\mathrm{cis}\left(\frac{3\pi}{2}\right)$

i) $z_1 \cdot z_2 = \left[2 \cdot 3 \ ; \ \frac{\pi}{2} + \frac{3\pi}{2}\right] = \left[6 \ ; \ 2\pi\right] = 6\,\mathrm{cis}(2\pi)$

$\Rightarrow z_1 \cdot z_2 = 6\left(\cos(2\pi) + i\sin(2\pi)\right) = 6(1 + 0i)$

$\Rightarrow \boxed{z_1 \cdot z_2 = 6}$

ii) $\frac{z_1}{z_2} = \left[\frac{r_1}{r_2} ; \theta_1 - \theta_2\right] = \left[\frac{2}{3} ; \frac{\pi}{2} - \frac{3\pi}{2}\right]$

$= \left[\frac{2}{3} ; -\pi\right] = \frac{2}{3}\left(\cos(-\pi) + i\sin(-\pi)\right)$

$= \frac{2}{3}(-1 + 0i) = -\frac{2}{3}$

$\Rightarrow \boxed{\frac{z_1}{z_2} = -\frac{2}{3}}$

d) $z_1 = -10 \quad , \quad z_2 = -4$

$\Rightarrow z_1 \cdot z_2 = 40$

$\frac{z_1}{z_2} = \frac{-10}{-4} = \frac{5}{2}$

**1.2.6** Calculer le module $|z|$ des nombres complexes suivants :

a) $z = 2 + 3\,i$

b) $z = 1 + i$

c) $z = 2\,i$

d) $z = -3\,i$

e) $z = -1/2 + \sqrt{3}/2\,i$

f) $z = 5$

g) $z = -6$

h) $z = 0$

i) $z = \cos t + \sin t\, i$

a) $z = 2 + 3i \Rightarrow |z| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$

b) $z = 1 + i \Rightarrow |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

c) $z = 2i \Rightarrow |z| = \sqrt{0^2 + 2^2} = 2$

d) $z = -3i \Rightarrow |z| = \sqrt{(-3)^2} = 3$

e) $z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \Rightarrow |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$

f) $z = 5 \Rightarrow |z| = 5$

g) $z = -6 \Rightarrow |z| = 6$

h) $z = 0 \Rightarrow |z| = \sqrt{0^2 + 0^2} = 0$

i) $z = \cos(t) + \sin(t)\, i \Rightarrow |z| = \sqrt{\cos^2(t) + \sin^2(t)} = 1$