Corrigé

Chapitre 4 : Puissances, racines, exponentielles et logarithmes

*Exercice 1.1.1 :

Simplifier les expressions suivantes :

a) $2^4 \cdot 3^4 = (2 \cdot 3)^4 = \underline{6^4}$

b) $2^3 \cdot (-3)^3 \cdot 4^3 = (2 \cdot (-3) \cdot 4)^3 = (-24)^3 = \underline{-24^3}$

c) $3^6 \cdot 5^6 = (3 \cdot 5)^6 = \underline{15^6}$

d) $5^0 \cdot 5^1 \cdot 5^2 \cdot \ldots \cdot 5^{10} = 5^{0+1+2+3+4+5+6+7+8+9+10} = \underline{5^{55}}$

e) $3^2 \cdot 5^2 \cdot 15^3 = (3 \cdot 5)^2 \cdot 15^3 = 15^2 \cdot 15^3 = 15^{2+3}$

$= \underline{15^5}$

f) $\dfrac{5^8}{5^6} = (5)^{8-6} = \underline{5^2}$

g) $\dfrac{5^6}{5^8} = (5)^{6-8} = 5^{-2} = \underline{\dfrac{1}{5^2}}$

h) $\left(-\dfrac{2}{3}\right)^5 = -\left(\dfrac{2}{3}\right)^5 = \underline{-\dfrac{2^5}{3^5}}$ (5 impair)

i) $\dfrac{7 \cdot 5^5 \cdot 7^0 \cdot 7}{7^3 \cdot 7^4} = \dfrac{7^{1+5+0+1}}{7^{3+4}} = \dfrac{7^7}{7^7} = \underline{1}$

* Exercice 1.1.2 : Simplifier les expressions suivantes :

a) $\left(2^2\right)^3 = 2^{2\cdot 3} = 2^6$

b) $2^{(2^3)} = 2^8$

c) $\left(\left(-4\right)^2\right)^4 = \left(\left(4\right)^2\right)^4 = 4^{2\cdot 4} = 4^8 = (2^2)^8$ (4 ← pair)

$= 2^{2\cdot 8} = 2^{16}$

d) $\left(\left(\frac{1}{3}\right)^3\right)^6 = \left(\frac{1}{3}\right)^{3\cdot 6} = \left(\frac{1}{3}\right)^{18} = \frac{1}{3^{18}}$

e) $\left(-\frac{2^4}{3^3}\right)^2 = \left(\frac{2^4}{3^3}\right)^2 = \frac{(2^4)^2}{(3^3)^2} = \frac{2^{4\cdot 2}}{3^{3\cdot 2}}$

$= \frac{2^8}{3^6}$

f) $\left(\frac{2}{3}\right)^3 \div \left(\frac{5}{3}\right)^3 = \frac{2^3}{3^3} \div \frac{5^3}{3^3} = \frac{2^3}{5^3} \cdot \frac{3^3}{5^3}$

$= \frac{2^3}{5^3}$

g) $4^2 \cdot 2^5 \cdot 8^2 = (2^2)^2 \cdot 2^5 \cdot (2^3)^2 = 2^{2\cdot 2} \cdot 2^5 \cdot 2^{3\cdot 2}$

$= 2^4 \cdot 2^5 \cdot 2^6 = 2^{4+5+6} = 2^{15}$

h) $\left(\frac{3}{4}\right)^4 \div \left(\frac{9}{8}\right)^4 = \frac{3^4}{4^4} \div \frac{9^4}{8^4} = \frac{3^4}{(2^2)^4} \div \frac{(3^2)^4}{(2^3)^4}$

$= \frac{3^4}{2^8} \div \frac{3^8}{2^{12}} = \frac{3^4}{2^8} \cdot \frac{2^{12}}{3^8} = \frac{3^4}{3^8} \cdot \frac{2^{12}}{2^8}$

$= (3)^{4-8} \cdot (2)^{12-8} = 3^{-4} \cdot 2^4 = \frac{2^4}{3^4}$

i) $\frac{(1 \cdot 9 \cdot 27 \cdot 81)^5}{3^{50}} = \frac{(3^1 \cdot 3^2 \cdot 3^3 \cdot 3^4)^5}{3^{50}} = \frac{(3^{1+2+3+4})^5}{3^{50}}$

$= \frac{(3^{10})^5}{3^{50}} = 3^{50-50} = 3^0 = 1$

*Exercice 1.1.3 : Calculer :

a) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

b) $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

c) $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

d) $\left(\frac{1}{4}\right)^{-1} = \left(\frac{4}{1}\right)^{1} = 4$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$

e) $\left(\frac{-1}{2}\right)^{-2} = \left(\frac{2}{-1}\right)^{2} = \frac{2^2}{(-1)^2} = \frac{4}{1} = 4$

f) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{3^3}{2^3} = \frac{27}{8}$

* Exercice 1.1.4 :

2^{11}	2^{-2}	2^{-3}	2^{8}
2^{0}	2^{6}	2^{5}	2^{3}
2^{4}	2	2^{2}	2^{7}
2^{-1}	2^{9}	2^{10}	2^{-4}

$2^{11+0+4-1} = 2^{14}$

Exemple : $2^{-1} \cdot 2^{9} \cdot 2^{10} \cdot 2^{-4} = 2^{-1+9+10-4} = 2^{14}$

* Exercice 2.1.5 : Simplifier les expressions suivantes :

a) $2^4 \cdot 2^{-2} \cdot 2 = 2^{4-2+1} = \underline{2^3}$

b) $(2^3)^{-5} = 2^{3\cdot(-5)} = \underline{2^{-15}}$

c) $\dfrac{5^3}{5^{-2}} = (5)^{3-(-2)} = (5)^{3+2} = \underline{5^5}$

d) $\left(\left(-1\right)^{-2}\right)^{-3} = \left(-1\right)^{(-2)\cdot(-3)} = (-1)^6 = \underline{1}$

e) $\left(2^{-1}\cdot 5^{-1}\right)^{-1} = \left((2\cdot 5)^{-1}\right)^{-1} = \left(10^{-1}\right)^{-1}$

$= 10^{(-1)\cdot(-1)} = 10^1 = \underline{10}$

f) $\left(\dfrac{11^{-2}}{11^8}\right)^{-5} = \left(\left(11\right)^{-2-8}\right)^{-5} = \left(\left(11\right)^{-10}\right)^{-5}$

$= \left(11\right)^{(-10)\cdot(-5)} = \underline{11^{50}}$

g) $7^{-3}\cdot\dfrac{49}{7^8}\cdot 7 = 7^{-3}\cdot\dfrac{7^2}{7^8}\cdot 7 = \dfrac{7^{-3+2+1}}{7^8}$

$= \dfrac{7^0}{7^8} = \dfrac{1}{7^8} = \underline{7^{-8}}$

h) $10'000\cdot\dfrac{100}{100'000}\cdot 10^{-3} = 10^4\cdot\dfrac{10^2}{10^5}\cdot 10^{-3} = \dfrac{10^{4+2-3}}{10^5}$

$$= \frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$$

i) $$\frac{1'280 \cdot 5^7 \cdot 125}{(0{,}2 \cdot 25)^3} = \frac{1'280 \cdot 5^7 \cdot 5^3}{5^3} = 1280 \cdot 5^7$$

$$= 1'280 \cdot 78125 = 100'000'000 = 10^8$$

* Exercice 1.1.6 : Simplifier les expressions suivantes et écrivez-les sans fraction :

a) $x^2 y z^3 \cdot 3xy \cdot 27x^3 z^5 = 3 \cdot 27 \cdot x^2 \cdot x \cdot x^3 \cdot y \cdot y \cdot z^3 \cdot z^5$

$= 3 \cdot 3^3 \cdot x^6 \cdot y^2 \cdot z^8 = 3^4 x^6 y^2 z^8$

b) $(2a^2b^3c)^4 = 2^4 \cdot a^{2 \cdot 4} \cdot b^{3 \cdot 4} \cdot c^4 = 2^4 \cdot a^8 \cdot b^{12} \cdot c^4$

$= 2^4 a^8 b^{12} c^4$

c) $$\left(\frac{2r^3}{s}\right)^2 \cdot \left(\frac{s}{r}\right)^3 = \frac{(2r^3)^2}{s^2} \cdot \frac{s^3}{r^3} = \frac{4r^6}{s^2} \cdot \frac{s^3}{r^3}$$

$$= \frac{4r^6}{r^3} \cdot \frac{s^3}{s^2} = 4r^{6-3} \cdot s^{3-2} = 4r^3 \cdot s$$

$= 2^2 r^3 s$

d) $\dfrac{(4x^2y^3)^5}{(2xy)^3} \div \dfrac{x^7}{(y^3)^4} = \dfrac{4^5 \cdot x^{10} \cdot y^{15}}{2^3 \cdot x^3 \cdot y^3} \div \dfrac{x^7}{y^{12}}$

$= \dfrac{(2^2)^5 \cdot x^{10-3} \cdot y^{15-3}}{2^3} \div \dfrac{x^7}{y^{12}} = \dfrac{2^{10} \cdot x^7 \cdot y^{12}}{2^3} \div \dfrac{x^7}{y^{12}}$

$= \dfrac{2^{10} \cdot \cancel{x^7} \cdot y^{12}}{2^3} \cdot \dfrac{y^{12}}{\cancel{x^7}} = 2^{10-3} \cdot y^{12} \cdot y^{12} = 2^7 \cdot y^{24}$

e) $(u^{-2}v^3)^{-3} = u^{(-2)(-3)} \cdot v^{(3)\cdot(-3)} = u^6 v^{-9}$

f) $\dfrac{8x^3y^{-5}}{4x^{-1}y^2} = 2 \cdot x^{3-(-1)} \cdot y^{-5-2} = 2x^4y^{-7}$

g) $\left(\dfrac{x}{3}\right)^{-2} \div \left(\dfrac{x}{9}\right)^{-3} = \left(\dfrac{3}{x}\right)^2 \div \left(\dfrac{9}{x}\right)^3$

$= \dfrac{3^2}{x^2} \cdot \dfrac{x^3}{9^3} = \dfrac{3^2}{x^2} \cdot \dfrac{x^3}{3^6} = \dfrac{3^2}{3^6} \cdot \dfrac{x^3}{x^2}$

$= 3^{-4} \cdot x$

h) $\left(\dfrac{9y^3(3y^2)^{-2}}{(y^{-4})^{-3}}\right)^5 = \dfrac{9^5 y^{15} (3y^2)^{-10}}{(y^{-4})^{-15}}$

$$= \frac{y^{10} \cdot y^{15}}{y^{60}} \cdot \frac{y^{-10}}{y^{-20}} = \frac{y^{-5}}{y^{60}} = y^{-5-60} = y^{-65}$$

* Exercice 2.1.7 : Calculer

a) $\sqrt{25} = \sqrt{5^2} = 5$

b) $\sqrt[3]{1000} = \sqrt[3]{10^3} = 10^{3/3} = 10$

c) $\sqrt[4]{625} = \sqrt[4]{5^4} = 5$

d) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$

e) $\sqrt[6]{729} = \sqrt[6]{3^6} = 3$

f) $\sqrt[3]{0,027} = \sqrt[3]{27 \cdot 10^{-3}} = \sqrt[3]{3^3 \cdot 10^{-3}} = 3^{3/3} \cdot 10^{-3/3}$

$= 3 \cdot 10^{-1} = \frac{3}{10} = 0,3$

g) $\sqrt[3]{0,125} = \sqrt[3]{125 \cdot 10^{-3}} = \sqrt[3]{5^3 \cdot 10^{-3}} = 5 \cdot 10^{-1}$

$= \frac{5}{10} = 0,5$

h) $\sqrt[3]{0,015625} = \sqrt[3]{15625 \cdot 10^{-6}} = \sqrt[3]{25^3 \cdot 10^{-6}}$

$= 25 \cdot 10^{-2} = \frac{25}{10^2} = 0,25$

i) $\sqrt{0} = 0$

j) $\sqrt[3]{0,000008} = \sqrt[3]{8 \cdot 10^{-6}} = \sqrt[3]{2^3 \cdot 10^{-6}}$

$= 2 \cdot 10^{-2} = \frac{2}{10^2} = 0,02$

* Exercice 1.1.8 : Simplifier les expressions suivantes :

a) $\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{2^2 \cdot 6} = 2\sqrt{6}$

b) $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{3^2 \cdot 2} = 3\sqrt{2}$

c) $\sqrt{243} = \sqrt{81 \cdot 3} = \sqrt{9^2 \cdot 3} = 9\sqrt{3}$

(ou $\sqrt{81 \cdot 3} = \sqrt{81} \cdot \sqrt{3} = 9 \cdot \sqrt{3}$)

d) $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$

e) $\sqrt{300} = \sqrt{100 \cdot 3} = \sqrt{10^2 \cdot 3} = 10\sqrt{3}$

f) $\sqrt{54} = \sqrt{9 \cdot 6} = \sqrt{3^2 \cdot 6} = 3\sqrt{6}$

g) $\sqrt{125} = \sqrt{25 \cdot 5} = \sqrt{5^2 \cdot 5} = 5\sqrt{5}$

h) $\sqrt{147} = \sqrt{49 \cdot 3} = \sqrt{7^2 \cdot 3} = 7\sqrt{3}$

i) $\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{4^2 \cdot 5} = 4\sqrt{5}$

j) $\sqrt{1000} = \sqrt{100 \cdot 10} = \sqrt{10^2 \cdot 10} = 10\sqrt{10}$

k) $\sqrt{250} = \sqrt{25 \cdot 10} = \sqrt{5^2 \cdot 10} = 5\sqrt{10}$

l) $\sqrt{7'000} = \sqrt{100 \cdot 70} = \sqrt{10^2 \cdot 70} = 10\sqrt{70}$

m) $3\sqrt{5} - 4\sqrt{20} + 5\sqrt{45} - 3\sqrt{80}$

$= 3\sqrt{5} - 4\sqrt{4 \cdot 5} + 5\sqrt{9 \cdot 5} - 3\sqrt{16 \cdot 5} = 3\sqrt{5} - 4 \cdot 2\sqrt{5} + 5 \cdot 3\sqrt{5} - 3 \cdot 4\sqrt{5}$

$= 3\sqrt{5} - 8\sqrt{5} + 15\sqrt{5} - 12\sqrt{5} = \sqrt{5}(3 - 8 + 15 - 12)$

$= -2\sqrt{5}$

n) $2\sqrt{40} - 2\sqrt{90} + \sqrt{4000} - 5\sqrt{10}$

$= 2\sqrt{4 \cdot 10} - 2\sqrt{9 \cdot 10} + \sqrt{400 \cdot 10} - 5\sqrt{10} = 2 \cdot 2\sqrt{10} - 2 \cdot 3\sqrt{10} + 20\sqrt{10} - 5\sqrt{10}$

$= 4\sqrt{10} - 6\sqrt{10} + 20\sqrt{10} - 5\sqrt{10} = \sqrt{10}(4 - 6 + 20 - 5) = 13\sqrt{10}$

* Exercice 4.1.9 : Effectuer et réduire :

a) $(9\sqrt{12} + 3)(\sqrt{3} + 8) = 9\sqrt{12} \cdot \sqrt{3} + 9 \cdot 8\sqrt{12} + 3\sqrt{3} + 3 \cdot 8$

$= 9\sqrt{12 \cdot 3} + 72\sqrt{12} + 3\sqrt{3} + 24 = 9\sqrt{6^2} + 72\sqrt{12} + 3\sqrt{3} + 24$

$= 9 \cdot 6 + 72\sqrt{4 \cdot 3} + 3\sqrt{3} + 24 = 78 + 72 \cdot 2\sqrt{3} + 3\sqrt{3}$

$= 78 + 144\sqrt{3} + 3\sqrt{3} = 78 + 147\sqrt{3}$

b) $(4\sqrt{3} + \sqrt{45})(\sqrt{5} - 2\sqrt{27})$

$= 4\sqrt{3}\cdot\sqrt{5} - 8\sqrt{3}\sqrt{27} + \sqrt{45}\sqrt{5} - 2\sqrt{45}\sqrt{27}$

$= 4\sqrt{15} - 8\sqrt{3\cdot 3^3} + \sqrt{45\cdot 5} - 2\sqrt{45\cdot 27}$

$= 4\sqrt{15} - 8\sqrt{3^4} + \sqrt{9\cdot 5\cdot 5} - 2\sqrt{9\cdot 5\cdot 3^3}$

$= 4\sqrt{15} - 8\cdot 9 + 3\cdot 5 - 2\cdot\sqrt{3^2\cdot 3^2\cdot 3\cdot 5} = 4\sqrt{15} - 72 + 15 - 2\cdot 3\cdot 3\sqrt{15}$

$= 4\sqrt{15} - 57 - 18\sqrt{15} = -57 - 14\sqrt{15}$

c) $\sqrt{3-2\sqrt{2}}\cdot\sqrt{3+2\sqrt{2}} = \sqrt{(3-2\sqrt{2})(3+2\sqrt{2})}$

$= \sqrt{9-(2\sqrt{2})^2} = \sqrt{9-8} = 1$

d) $(\sqrt{3}+1)^4 = \left((\sqrt{3}+1)^2\right)^2 = \left((\sqrt{3})^2 + 1^2 + 2\sqrt{3}\right)^2$

$= (3+1+2\sqrt{3})^2 = (4+2\sqrt{3})^2 = 4^2 + (2\sqrt{3})^2 + 2\cdot 4\cdot 2\sqrt{3}$

$= 16 + 12 + 16\sqrt{3} = 28 + 16\sqrt{3}$

* Exercice 1.1.10 Simplifier les expressions suivantes :

a) $\sqrt[3]{\sqrt{7}} = \sqrt[3]{7^{1/2}} = 7^{\left(\frac{1}{2}/3\right)} = 7^{\frac{1}{6}} = \sqrt[6]{7}$

(ou $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m.n]{a}$ ex: $\sqrt[3]{\sqrt{7}} = \sqrt[3.2]{7} = \sqrt[6]{7}$)

b) $\sqrt[3]{2^{18} \cdot 5^{12} \cdot 3^3} = 2^{\frac{18}{3}} \cdot 5^{\frac{12}{3}} \cdot 3^{\frac{3}{3}} = 2^6 \cdot 5^4 \cdot 3$

$= 64 \cdot 625 \cdot 3 = 120'000$

c) $\sqrt[4]{64} \cdot \sqrt[4]{4} = \sqrt[4]{64 \cdot 4} = \sqrt[4]{4^3 \cdot 4} = \sqrt[4]{4^4} = 4$

d) $\sqrt[5]{3^{15}} = 3^{15/5} = 3^3 = 27$

e) $\left(\sqrt[8]{\sqrt[4]{\sqrt{2}}}\right)^{128} = \left(\sqrt[8.4.2]{2}\right)^{128}$

$= \left(\sqrt[64]{2}\right)^{128} = \sqrt[64]{2^{128}} = 2^{\frac{128}{64}} = 2^2 = 4$

f) $\sqrt{3\sqrt{3}} = \sqrt{3 \cdot 3^{1/2}} = \sqrt{3^{1+\frac{1}{2}}} = \sqrt{3^{3/2}}$

$= 3^{\frac{3}{2}/2} = 3^{\frac{3}{4}} = \sqrt[4]{3^3} = \sqrt[4]{27}$

(ou $\sqrt{3\sqrt{3}} = \sqrt{\sqrt{9 \cdot 3}} = \sqrt{\sqrt{27}} = \sqrt[4]{27}$)

g) $\sqrt[3]{5\sqrt{5\sqrt{5}}} = \sqrt[3]{5\sqrt{\sqrt{5^2 \cdot 5}}} = \sqrt[3]{5\sqrt{\sqrt{5^3}}}$

$= \sqrt[3]{5\sqrt[4]{5^3}} = \sqrt[3]{\sqrt[4]{5^4 \cdot 5^3}} = \sqrt[3]{\sqrt[4]{5^7}}$

$= \sqrt[12]{5^7} = \sqrt[12]{78125}$

h) $\sqrt{2\sqrt[3]{2}} = \sqrt{\sqrt[3]{2^3 \cdot 2}} = \sqrt{\sqrt[3]{2^4}} = \sqrt[6]{2^4}$

$= 2^{\frac{4}{6}} = 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}$

i) $\sqrt[3]{3\sqrt[3]{3^4\sqrt[3]{3^6}}} = \sqrt[3]{3\sqrt[3]{3^4 \cdot 3^{6/3}}} = \sqrt[3]{3\sqrt[3]{3^4 \cdot 3^2}}$

$= \sqrt[3]{3\sqrt[3]{3^6}} = \sqrt[3]{3 \cdot 3^{6/3}} = \sqrt[3]{3 \cdot 3^2}$

$= \sqrt[3]{3^3} = 3^{3/3} = 3$

(ou $\sqrt[3]{3\sqrt[3]{3^4\sqrt[3]{3^6}}} = \sqrt[3]{3\sqrt[3]{3^3 \cdot 3^4\sqrt[3]{3^6}}}$

$= \sqrt[9]{\sqrt[3]{3^{21} \cdot 3^6}} = \sqrt[27]{3^{27}} = 3$)

j) $\sqrt[3]{2\sqrt[6]{\frac{2^{14}}{\sqrt[3]{2^6}}}} = \sqrt[3]{2\sqrt[6]{\frac{2^{14}}{2^2}}} = \sqrt[3]{2\sqrt[6]{2^{12}}}$

$$= \sqrt[3]{2 \cdot 2^2} = \sqrt[3]{2^3} = 2^{3/3} = 2$$

* Exercice 2.1.11 Simplifier les expressions suivantes :

a) $\sqrt[5]{a^3} \cdot \left(\sqrt[5]{a}\right)^2 = \sqrt[5]{a^3} \cdot \sqrt[5]{a^2}$

$\left(\left(\sqrt[n]{a}\right)^p = \sqrt[n]{a^p} \right)$

$$= \sqrt[5]{a^3 \cdot a^2} = \sqrt[5]{a^5} = a$$

b) $\sqrt[3]{a} \cdot \left(\sqrt[3]{a}\right)^2 = \sqrt[3]{a} \cdot \sqrt[3]{a^2} = \sqrt[3]{a \cdot a^2} = a$

$\left(\text{ou } \sqrt[3]{a} \cdot \left(\sqrt[3]{a}\right)^2 = a^{\frac{1}{3}} \cdot a^{\frac{2}{3}} = a^{\frac{1}{3}+\frac{2}{3}} = a^{3/3} = a \right)$

c) $\sqrt[5]{a^3} \cdot \left(\sqrt[5]{a^2}\right)^6 = \sqrt[5]{a^3} \cdot \sqrt[5]{(a^2)^6} = \sqrt[5]{a^3} \cdot \sqrt[5]{a^{12}}$

$$= \sqrt[5]{a^3 \cdot a^{12}} = \sqrt[5]{a^{15}} = a^{15/5} = a^3$$

d) $\sqrt[4]{a^3} \cdot \sqrt[3]{a^4} = a^{\frac{3}{4}} \cdot a^{\frac{4}{3}} = a^{\frac{3}{4}+\frac{4}{3}} = a^{\frac{9+16}{12}}$

$$= a^{\frac{25}{12}} = \sqrt[12]{a^{25}}$$

e) $\sqrt{a} \cdot \sqrt[5]{a^3} \cdot \left(\sqrt[10]{a}\right)^4 = a^{\frac{1}{2}} \cdot a^{\frac{3}{5}} \cdot a^{\frac{4}{10}}$ $\quad a^{\frac{4}{10}} = a^{2/5}$

$= a^{\frac{1}{2}+\frac{3}{5}+\frac{2}{5}} = a^{\frac{5+6+4}{10}} = a^{\frac{15}{10}} = a^{\frac{3}{2}}$

$= \sqrt{a^3}$

f) $\sqrt[3]{a} \cdot \sqrt[4]{a^3} \cdot \sqrt[6]{a} = a^{\frac{1}{3}} \cdot a^{\frac{3}{4}} \cdot a^{\frac{1}{6}}$

$= a^{\frac{1}{3}+\frac{3}{4}+\frac{1}{6}} = a^{\frac{4+9+2}{12}} = a^{\frac{15}{12}} = a^{\frac{5}{4}} = \sqrt[4]{a^5}$

g) $\sqrt{\sqrt[3]{a}} = \sqrt[6]{a}$

h) $\left(\sqrt[10]{\sqrt[5]{a}}\right)^{15} = \left(\sqrt[50]{a}\right)^{15}$

$= \sqrt[50]{a^{15}} = a^{\frac{15}{50}} = a^{\frac{3}{10}} = \sqrt[10]{a^3}$

i) $\dfrac{\sqrt[3]{a^4}}{\sqrt{a}} = \dfrac{a^{\frac{4}{3}}}{a^{\frac{1}{2}}} = a^{\left(\frac{4}{3}-\frac{1}{2}\right)} = a^{\frac{8-3}{6}} = a^{\frac{5}{6}}$

$= \sqrt[6]{a^5}$

j) $\dfrac{\sqrt[6]{a^5}}{\sqrt[4]{a^3}} = \dfrac{a^{\frac{5}{6}}}{a^{\frac{3}{4}}} = a^{\left(\frac{5}{6}-\frac{3}{4}\right)} = a^{\frac{10-9}{12}}$

$= a^{\frac{1}{12}} = \sqrt[12]{a}$

k) $\dfrac{\sqrt{a} \cdot \sqrt[3]{a}}{\sqrt[4]{a^3}} = \dfrac{a^{\frac{1}{2}} \cdot a^{\frac{1}{3}}}{a^{\frac{3}{4}}} = \dfrac{a^{\left(\frac{1}{2}+\frac{1}{3}\right)}}{a^{\frac{3}{4}}} = \dfrac{a^{\frac{3+2}{6}}}{a^{\frac{3}{4}}}$

$= \dfrac{a^{\frac{5}{6}}}{a^{\frac{3}{4}}} = a^{\frac{5}{6}} \cdot a^{-\frac{3}{4}} = a^{\left(\frac{5}{6}-\frac{3}{4}\right)} = a^{\frac{10-9}{12}}$

$= a^{\frac{1}{12}} = \sqrt[12]{a}$

(ou $\dfrac{\sqrt{a} \cdot \sqrt[3]{a}}{\sqrt[4]{a^3}} = \dfrac{a^{\frac{1}{2}} \cdot a^{\frac{1}{3}}}{a^{\frac{3}{4}}} = a^{\frac{1}{2}} \cdot a^{\frac{1}{3}} \cdot a^{-\frac{3}{4}} = \sqrt[12]{a}$)

l) $\dfrac{a^3}{\sqrt[3]{a^5} \cdot \sqrt[6]{a}} = \dfrac{a^3}{a^{\frac{5}{3}} \cdot a^{\frac{1}{6}}} = a^3 \cdot a^{-\frac{5}{3}} \cdot a^{-\frac{1}{6}}$

$= a^{\left(3-\frac{5}{3}-\frac{1}{6}\right)} = a^{\left(\frac{18-10-1}{6}\right)} = a^{\frac{7}{6}} = \sqrt[6]{a^7}$

* Exercice 2.1.12 : Rendre rationnel les dénominateurs et simplifier les expressions.

"Rendre rationnel le dénominateur d'une fraction, c'est supprimer la racine du dénominateur de cette fraction"

a) $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) $\frac{2}{\sqrt[4]{5}} = \frac{2\cdot\left(\sqrt[4]{5}\right)^3}{\sqrt[4]{5}\cdot\left(\sqrt[4]{5}\right)^3} = \frac{2\sqrt[4]{5^3}}{\sqrt[4]{5\cdot 5^3}} = \frac{2\cdot\sqrt[4]{125}}{\sqrt[4]{5^4}}$

$= \frac{2\sqrt[4]{125}}{5}$

c) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{3}}{3}$

d) $\frac{1}{2+\sqrt{3}} = \frac{(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3}$

$= 2-\sqrt{3}$

e) $\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2\cdot(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{2(\sqrt{5}-\sqrt{3})}{5-3} = \frac{2(\sqrt{5}-\sqrt{3})}{2} = \sqrt{5}-\sqrt{3}$

f) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}} = \frac{1}{\sqrt[3]{3}-\sqrt[3]{2}}\cdot\frac{\left(\sqrt[3]{3}\right)^2+\sqrt[3]{3\cdot 2}+\left(\sqrt[3]{2}\right)^2}{\left(\sqrt[3]{3}\right)^2+\sqrt[3]{3\cdot 2}+\left(\sqrt[3]{2}\right)^2}$

$= \sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}$

* Exercice 1.1.13 Écrire à l'aide d'exposants rationnels :

a) $\sqrt[3]{5^2} = a^{\frac{2}{3}}$

b) $\sqrt[10]{7} = 7^{\frac{1}{10}}$

c) $-\sqrt[8]{7^2} = -7^{\frac{2}{8}} = -7^{\frac{1}{4}}$

d) $\sqrt{2} = 2^{\frac{1}{2}}$

e) $\frac{1}{\sqrt{3}} = \frac{1}{3^{\frac{1}{2}}} = 3^{-\frac{1}{2}}$

f) $\frac{8}{\sqrt[7]{4^3}} = \frac{2^3}{\sqrt[7]{(2^2)^3}} = \frac{2^3}{\sqrt[7]{2^6}} = \frac{2^3}{2^{6/7}}$

$= 2^3 \cdot 2^{-\frac{6}{7}} = 2^{3-\frac{6}{7}} = 2^{\frac{21-6}{7}} = 2^{\frac{15}{7}}$

g) $\sqrt[4]{5} = 5^{\frac{1}{4}}$

h) $\sqrt[7]{3^7} = 3^{\frac{7}{7}} = 3$

* **Exercice 2.1.14** Écrire à l'aide de racines et d'exposants entiers positifs.

a) $7^{\frac{3}{2}} = \sqrt{7^3}$

b) $3^{\frac{2}{5}} = \sqrt[5]{3^2}$

c) $64^{\frac{3}{2}} = \sqrt{64^3}$

d) $-11^{0,25} = -11^{\frac{1}{4}} = -\sqrt[4]{11}$

e) $36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{\sqrt{36}}$

f) $8^{-\frac{7}{5}} = \frac{1}{8^{\frac{7}{5}}} = \frac{1}{\sqrt[5]{8^7}}$

g) $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}}$

h) $(-5)^{0,5} = (-3)^{\frac{1}{2}} = \sqrt{-5}$ impossible !

* **Exercice 2.1.15** Calculer sans l'aide de la machine

a) $\sqrt[4]{16^3} = \sqrt[4]{(2^4)^3} = \sqrt[4]{2^{12}} = 2^{\frac{12}{4}} = 2^3 = 8$

b) $\left(5 + 16^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(5 + \sqrt{16}\right)^{\frac{1}{2}} = (5+4)^{\frac{1}{2}}$

$= 9^{\frac{1}{2}} = \sqrt{9} = 3$

c) $4 \cdot 25^{\frac{3}{2}} = 4 \cdot (5^2)^{\frac{3}{2}} = 4 \cdot 5^3 = 4 \cdot 125 = 500$

d) $(4 \cdot 25)^{\frac{3}{2}} = (2^2 \cdot 5^2)^{\frac{3}{2}} = ((2 \cdot 5)^2)^{\frac{3}{2}}$

$= (10^2)^{\frac{3}{2}} = 10^{2 \cdot \frac{3}{2}} = 10^3 = 1000$

e) $19 - 27^{\frac{1}{3}} = 19 - \sqrt[3]{27} = 19 - \sqrt[3]{3^3} = 19 - 3 = 16$

f) $(19 - 27)^{\frac{1}{3}} = (-8)^{\frac{1}{3}} = \sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

g) $(-32)^{\frac{1}{5}} = ((-2)^5)^{\frac{1}{5}} = (-2)^{5 \cdot \frac{1}{5}} = -2$

h) $(32)^{-\frac{1}{5}} = (2^5)^{-\frac{1}{5}} = 2^{-1} = \frac{1}{2}$

* Exercice 2.1.16 Calculer

a) $8^{\frac{2}{3}} + 16^{\frac{1}{2}} + 27^{\frac{2}{3}} + 81^{\frac{1}{4}} - 125^{\frac{1}{3}} - 1000^{\frac{2}{3}}$

$= (2^3)^{\frac{2}{3}} + (2^4)^{\frac{1}{2}} + (3^3)^{\frac{2}{3}} + (3^4)^{\frac{1}{4}} - (5^3)^{\frac{1}{3}} - (10^3)^{\frac{2}{3}}$

$= 2^2 + 2^2 + 3^2 + 3 - 5 - 10^2 = 4 + 4 + 9 + 3 - 5 - 100$

$= -85$

b) $\left(3 \cdot 32^{\frac{1}{3}} + 3 \cdot 108^{\frac{1}{3}} - 256 \cdot 2^{\frac{2}{3}}\right) \cdot 2^{\frac{1}{3}}$

$= \left(3 \cdot (2^5)^{\frac{1}{3}} + 3 \cdot (4 \cdot 3^3)^{\frac{1}{3}} - 2^8 \cdot 2^{\frac{2}{3}}\right) \cdot 2^{\frac{1}{3}}$

$= \left(3 \cdot 2^{\frac{5}{3}} + 3 \cdot (2^2)^{\frac{1}{3}} \cdot 3^{3 \cdot \frac{1}{3}} - 2^{8 + \frac{2}{3}}\right) \cdot 2^{\frac{1}{3}}$

$$= \left(3 \cdot 2^{\frac{5}{3}} + 3 \cdot 2^{\frac{2}{3}} \cdot 3 - 2^{\frac{26}{3}} \right) \cdot 2^{\frac{1}{3}}$$

$$= \left(3 \cdot 2^{\frac{5}{3}} + 9 \cdot 2^{\frac{2}{3}} - 2^{\frac{26}{3}} \right) \cdot 2^{\frac{1}{3}}$$

$$= 3 \cdot 2^{\frac{5}{3}} \cdot 2^{\frac{1}{3}} + 9 \cdot 2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} - 2^{\frac{26}{3}} \cdot 2^{\frac{1}{3}}$$

$$= 3 \cdot 2^{\frac{6}{3}} + 9 \cdot 2^{\frac{3}{3}} - 2^{\frac{27}{3}} = 3 \cdot 2^{2} + 9 \cdot 2 - 2^{9}$$

$$= 3 \cdot 4 + 18 - 512 = 12 + 18 - 512 = -482$$

c) $\left(3 \cdot 2^{0,25} + 2 \cdot 32^{0,25} - 8^{0,75} \right) \cdot 8^{0,25}$

$$= \left(3 \cdot 2^{\frac{1}{4}} + 2 \cdot 32^{\frac{1}{4}} - 8^{\frac{3}{4}} \right) \cdot 8^{\frac{1}{4}}$$

$$= \left(3 \cdot 2^{\frac{1}{4}} + 2 \cdot \left(2^{5}\right)^{\frac{1}{4}} - \left(2^{3}\right)^{\frac{3}{4}} \right) \cdot \left(2^{3}\right)^{\frac{1}{4}}$$

$$= \left(3 \cdot 2^{\frac{1}{4}} + 2 \cdot 2^{\frac{5}{4}} - 2^{\frac{9}{4}} \right) \cdot 2^{\frac{3}{4}}$$

$$= 3 \cdot 2^{\frac{1}{4}} \cdot 2^{\frac{3}{4}} + 2 \cdot 2^{\frac{5}{4}} \cdot 2^{\frac{3}{4}} - 2^{\frac{9}{4}} \cdot 2^{\frac{3}{4}}$$

$$= 3 \cdot 2^{\frac{1}{4}+\frac{3}{4}} + 2^{1+\frac{5}{4}+\frac{3}{4}} - 2^{\frac{9}{4}+\frac{3}{4}}$$

$$= 3 \cdot 2 + 2^{1+2} - 2^{3} = 6 + 8 - 8 = 6$$

d) $$\frac{16^{\frac{1}{3}} - 4 \cdot 128^{\frac{1}{3}} + 3 \cdot 250^{\frac{1}{3}}}{2^{\frac{1}{3}}}$$

$$= \frac{\left(2^4\right)^{\frac{1}{3}} - 2^2 \cdot \left(2^7\right)^{\frac{1}{3}} + 3 \cdot \left(2 \cdot 5^3\right)^{\frac{1}{3}}}{2^{\frac{1}{3}}}$$

$$= \frac{2^{\frac{4}{3}} - 2^2 \cdot 2^{\frac{7}{3}} + 3 \cdot 2^{\frac{1}{3}} \cdot 5^{3 \cdot \frac{1}{3}}}{2^{\frac{1}{3}}}$$

$$= \left(2^{\frac{4}{3}} - 2^2 \cdot 2^{\frac{7}{3}} + 3 \cdot 2^{\frac{1}{3}} \cdot 5\right) \cdot 2^{-\frac{1}{3}}$$

$$= 2^{\frac{4}{3}} \cdot 2^{-\frac{1}{3}} - 2^2 \cdot 2^{\frac{7}{3}} \cdot 2^{-\frac{1}{3}} + 3 \cdot 2^{\frac{1}{3}} \cdot 2^{-\frac{1}{3}} \cdot 5$$

$$= 2^{\frac{4}{3} - \frac{1}{3}} - 2^{2 + \frac{7}{3} - \frac{1}{3}} + 3 \cdot 2^{\frac{1}{3} - \frac{1}{3}} \cdot 5$$

$$= 2^{\frac{3}{3}} - 2^{2 + \frac{6}{3}} + 3 \cdot 2^0 \cdot 5$$

$$= 2^1 - 2^4 + 3 \cdot 5$$

$$= 2 - 16 + 15 = 1$$

4.1.17 Simplifier les expressions suivantes et écrivez-les sans fraction :

a) $u^{4/3}u^{-3/2}u^{1/6}$

b) $(a^{-2/3}b^{-1}c^{2})^{-3/2}\cdot(a^{-1/2}b^{1/3}c)^{-2}$

c) $\left(\dfrac{x^{-2/3}y^{3/4}}{x^{5/2}y^{2/3}}\right)^{1/5}\div\left(\dfrac{x^{4}y^{-2}}{x^{1/3}y^{-2/5}}\right)^{2/3}$

a) $u^{\frac{4}{3}}\,u^{-\frac{3}{2}}\,u^{\frac{1}{6}} = u^{\frac{4}{3}-\frac{3}{2}+\frac{1}{6}} = u^{\frac{8-9+1}{6}} = u^{0} = 1$ si $u \neq 0$

b) $\left(a^{-2/3}\,b^{-1}\,c^{2}\right)^{-3/2}\cdot\left(a^{-1/2}\,b^{1/3}\,c\right)^{-2}$

$= a^{(-\frac{2}{3})(-\frac{3}{2})}\,b^{(-1)(-3/2)}\,c^{2(-\frac{3}{2})}\cdot a^{(-\frac{1}{2})(-2)}\,b^{(1/3)(-2)}\,c^{-2}$

$= a\,b^{3/2}\,c^{-3}\cdot a\,b^{2/3}\,c^{-2} = a^{2}\,b^{\frac{3}{2}-\frac{2}{3}}\,c^{-5} = a^{2}\,b^{5/6}\,c^{-5}$

c) $\left(\dfrac{x^{-2/3}\,y^{3/4}}{x^{5/2}\,y^{2/3}}\right)^{1/5}\div\left(\dfrac{x^{4}\,y^{-2}}{x^{1/3}\,y^{-2/5}}\right)^{2/3}$

$= \left(x^{-\frac{2}{3}-\frac{5}{2}}\,y^{\frac{3}{4}-\frac{2}{3}}\right)^{1/5}\div\left(x^{4-\frac{1}{3}}\,y^{-2+\frac{2}{5}}\right)^{2/3}$

$= \left(x^{\frac{-4-15}{6}}\,y^{\frac{9-8}{12}}\right)^{1/5}\div\left(x^{\frac{12-1}{3}}\,y^{\frac{-10+2}{5}}\right)^{2/3}$

$= \left(x^{-\frac{19}{6}}\,y^{\frac{1}{12}}\right)^{1/5}\div\left(x^{\frac{11}{3}}\,y^{-\frac{8}{5}}\right)^{2/3}$

$= \left(x^{-\frac{19}{6}\cdot\frac{1}{5}}\,y^{\frac{1}{12}\cdot\frac{1}{5}}\right)\div\left(x^{\frac{11}{3}\cdot\frac{2}{3}}\,y^{-\frac{8}{5}\cdot\frac{2}{3}}\right)$

$= \left(x^{-\frac{19}{30}}\,y^{\frac{1}{60}}\right)\div\left(x^{\frac{22}{9}}\,y^{-\frac{16}{15}}\right)$

$$= \frac{x^{-\frac{19}{30}} \, y^{\frac{1}{60}}}{x^{\frac{22}{9}} \, y^{-\frac{16}{15}}} = x^{-\frac{19}{30} - \frac{22}{9}} \, y^{\frac{1}{60} + \frac{16}{15}}$$

$$= x^{\frac{-57 - 220}{90}} \, y^{\frac{1 + 64}{60}} = x^{-\frac{277}{90}} \, y^{\frac{65}{60}}$$

$$= \boxed{x^{-\frac{277}{90}} \, y^{\frac{13}{12}}}$$