2M

Corrigé

Analyse : Limites de fonctions

2.4.2 Calculer les limites suivantes :

a) $\lim\limits_{x\to 1}(4x^3 - 2x^2 + x - 1)$

b) $\lim\limits_{x\to -2}(x^2 - 5x)$

c) $\lim\limits_{x\to 0}\dfrac{x + 3x^2}{x + 1}$

d) $\lim\limits_{x\to 4}(-5)$

e) $\lim\limits_{x\to 3}\sqrt{x^2 - 5}$

f) $\lim\limits_{x\to 1}\dfrac{x^2 + 2x + 1}{x^3 + x^2 + x}$

g) $\lim\limits_{x\to \frac{\pi}{2}}\cos\left(x + \dfrac{\pi}{2}\right)$

h) $\lim\limits_{x\to 0}\dfrac{\sin(x) + 1}{2 - \tan(x)}$

a) $\lim\limits_{x\to 1}\left(4x^3 - 2x^2 + x - 1\right) = 4\cdot(1)^3 - 2\cdot(1)^2 + 1 - 1 = 4 - 2 + 1 - 1 = \underline{2}$

b) $\lim\limits_{x\to -2}\left(x^2 - 5x\right) = (-2)^2 - 5(-2) = 4 + 10 = \underline{14}$

c) $\lim\limits_{x\to 0}\dfrac{x + 3x^2}{x+1} = \dfrac{0 + 3\cdot 0}{0+1} = \underline{0}$

d) $\lim\limits_{x\to 4}(-5) = \underline{-5}$

e) $\lim\limits_{x\to 3}\sqrt{x^2 - 5} = \sqrt{9-5} = \sqrt{4} = \underline{2}$

f) $\lim\limits_{x\to 1}\dfrac{x^2 + 2x + 1}{x^3 + x^2 + x} = \dfrac{1 + 2 + 1}{1 + 1 + 1} = \underline{\dfrac{4}{3}}$

2.6.3 Calculer les limites suivantes :

a) $\lim\limits_{x\to 3} \dfrac{x-3}{2x-6}$ b) $\lim\limits_{x\to 0} \dfrac{100x^2}{x}$

c) $\lim\limits_{x\to 2} \dfrac{(x-2)(x+1)}{(x+4)(x-2)}$ d) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1}$

e) $\lim\limits_{x\to 5} \dfrac{x^2+2x-15}{x^2+8x+15}$ f) $\lim\limits_{x\to 4} \dfrac{x^2-4x}{x^2-16}$

g) $\lim\limits_{x\to 1} \dfrac{x^2-x}{x^3-4x^2-7x+10}$ h) $\lim\limits_{x\to 1} \dfrac{2x^3-3x^2+1}{x^2+3x-4}$

a) $\lim\limits_{x\to 3} \dfrac{x-3}{2x-6} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x\to 3} \dfrac{\cancel{x-3}}{2\cancel{(x-3)}} = \lim\limits_{x\to 3} \dfrac{1}{2} = \dfrac{1}{2}$

b) $\lim\limits_{x\to 0} \dfrac{100x^2}{x} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x\to 0} \dfrac{100\,x^{\cancel{2}}}{\cancel{x}} = \lim\limits_{x\to 0} \dfrac{100x}{1} = 0$

c) $\lim\limits_{x\to 2} \dfrac{(x\cancel{-2})(x+1)}{(x+4)(x\cancel{-2})} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x\to 2} \dfrac{x+1}{x+4} = \dfrac{3}{6} = \dfrac{1}{2}$

d) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \lim\limits_{x\to 1} x+1 = 2$

e) $\lim\limits_{x\to 5} \dfrac{x^2+2x-15}{x^2+8x+15} = \dfrac{25+10-15}{25+40+15} = \dfrac{20}{80} = \dfrac{2}{8} = \dfrac{1}{4}$

f) $\lim\limits_{x\to 4} \dfrac{x^2-4x}{x^2-16} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x\to 4} \dfrac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} = \lim\limits_{x\to 4} \dfrac{x}{x+4} = \dfrac{4}{8} = \dfrac{1}{2}$

g) $\lim\limits_{x\to 1} \dfrac{x^2-x}{x^3-4x^2-7x+10} \overset{\text{"}\frac{0}{0}\text{" fi}}{=}$

$*$ $x^2 - x = x(x-1)$

$x^3-4x^2-7x+10 \Rightarrow$ Horner

	1	-4	-7	10
1		1	-3	-10
	1	-3	-10	0

$\Rightarrow x^3-4x^2-7x+10 = (x-1)(x^2-3x-10)$

$$\Rightarrow \lim_{x \to 1} \frac{x^2 - x}{x^3 - 4x^2 - 7x + 10} = \lim_{x \to 1} \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x^2 - 3x - 10)}$$

$$= \lim_{x \to 1} \frac{x}{x^2 - 3x - 10} = \frac{1}{1 - 3 - 10} = \frac{1}{-12} = -\frac{1}{12}$$

h) $\lim_{x \to 1} \frac{2x^3 - 3x^2 + 1}{x^2 + 3x - 4}$ "$\frac{0}{0}$" FI $\Rightarrow$ il faut lever la forme indéterminée !

* Horner

	2	−3	0	1
1		2	−1	−1
	2	−1	−1	0

$$\Rightarrow 2x^3 - 3x^2 + 1 = (x-1)(2x^2 - x - 1)$$

* $x^2 + 3x - 4 = (x+4)(x-1)$

d'où $$\lim_{x \to 1} \frac{2x^3 - 3x^2 + 1}{x^2 + 3x - 4} = \lim_{x \to 1} \frac{\cancel{(x-1)}(2x^2 - x - 1)}{(x+4)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{2x^2 - x - 1}{x + 4} = \frac{2 \cdot (1)^2 - 1 - 1}{1 + 4} = \frac{2 - 2}{5} = 0$$

2.4.4 Calculer les limites suivantes :

a) $\lim\limits_{x\to 16} \dfrac{x-16}{\sqrt{x}-4}$

b) $\lim\limits_{x\to 25} \dfrac{\sqrt{x}-5}{x-25}$

c) $\lim\limits_{h\to 0} \dfrac{4-\sqrt{16+h}}{h}$

d) $\lim\limits_{x\to 0} \dfrac{x^2}{\sqrt{x^2+1}-1}$

e) $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{2x-1}-3}$

f) $\lim\limits_{t\to 2} \dfrac{1+\sqrt{t-2}}{t}$

g) $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x+1}-\sqrt{2x-1}}$

h) $\lim\limits_{x\to -2} \dfrac{x+\sqrt{x+6}}{x+\sqrt{2-x}}$

a) $$\lim_{x\to 16} \frac{x-16}{\sqrt{x}-4} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{x\to 16} \frac{(x-16)(\sqrt{x}+4)}{(\sqrt{x}-4)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{\cancel{(x-16)}(\sqrt{x}+4)}{\cancel{x-16}}$$

$$= \lim_{x\to 16} (\sqrt{x}+4) = \sqrt{16}+4 = \underline{8}$$

b) $$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{x\to 25} \frac{(\sqrt{x}-5)(\sqrt{x}+5)}{(x-25)(\sqrt{x}+5)} = \lim_{x\to 25} \frac{\cancel{(x-25)}}{\cancel{(x-25)}(\sqrt{x}+5)}$$

$$= \lim_{x\to 25} \left(\frac{1}{\sqrt{x}+5}\right) = \frac{1}{5+5} = \underline{\frac{1}{10}}$$

c) $$\lim_{h\to 0} \frac{4-\sqrt{16+h}}{h} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{h\to 0} \frac{(4-\sqrt{16+h})(4+\sqrt{16+h})}{h(4+\sqrt{16+h})}$$

$$= \lim_{h\to 0} \frac{16-(\sqrt{16+h})^2}{h(4+\sqrt{16+h})} = \lim_{h\to 0} \frac{16-(16+h)}{h(4+\sqrt{16+h})}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(4+\sqrt{16+h})} = \lim_{h\to 0} \frac{-1}{4+\sqrt{16+h}} = \frac{-1}{4+4} = \underline{-\frac{1}{8}}$$

d) $$\lim_{x\to 0} \frac{x^2}{\sqrt{x^2+1}-1} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{x\to 0} \frac{x^2(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1})^2-1} = \lim_{x\to 0} \frac{x^2(\sqrt{x^2+1}+1)}{x^2+1-1}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}\left(\sqrt{x^2+1} + 1\right)}{\cancel{x^2}} = \lim_{x \to 0} \frac{\sqrt{x^2+1} + 1}{1} = \frac{2}{1} = \underline{2}$$

e) $$\lim_{x \to 5} \frac{x-5}{\sqrt{2x-1} - 3} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{x \to 5} \frac{(x-5)\left(\sqrt{2x-1}+3\right)}{\left(\sqrt{2x-1}-3\right)\left(\sqrt{2x-1}+3\right)}$$

$$= \lim_{x \to 5} \frac{(x-5)\left(\sqrt{2x-1}+3\right)}{\left(\sqrt{2x-1}\right)^2 - (3)^2} = \lim_{x \to 5} \frac{(x-5)\left(\sqrt{2x-1}+3\right)}{(2x-1) - 9}$$

$$= \lim_{x \to 5} \frac{(x-5)\left(\sqrt{2x-1}+3\right)}{2x-10} = \lim_{x \to 5} \frac{\cancel{(x-5)}\left(\sqrt{2x-1}+3\right)}{2\cancel{(x-5)}}$$

$$= \lim_{x \to 5} \frac{\sqrt{2x-1}+3}{2} = \frac{\sqrt{2 \cdot 5 - 1}+3}{2} = \frac{6}{2} = \underline{3}$$

f) $$\lim_{t \to 2} \frac{1 + \sqrt{t-2}}{t} = \frac{1 + \sqrt{2-2}}{2} = \underline{\frac{1}{2}}$$

g) $$\lim_{x \to 2} \frac{x-2}{\sqrt{x+1} - \sqrt{2x-1}} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{x \to 2} \frac{(x-2)\left(\sqrt{x+1} + \sqrt{2x-1}\right)}{\left(\sqrt{x+1} - \sqrt{2x-1}\right)\left(\sqrt{x+1} + \sqrt{2x-1}\right)}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}\left(\sqrt{x+1} + \sqrt{2x-1}\right)}{-\cancel{(x-2)}} = \lim_{x \to 2} \frac{\sqrt{x+1} + \sqrt{2x-1}}{-1} = \frac{2\sqrt{3}}{-1} = \underline{-2\sqrt{3}}$$

h) $$\lim_{x \to -2} \frac{x + \sqrt{x+6}}{x + \sqrt{2-x}} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim_{x \to -2} \frac{\left(x + \sqrt{x+6}\right)\left(x - \sqrt{2-x}\right)}{\left(x + \sqrt{2-x}\right)\left(x - \sqrt{2-x}\right)}$$

$$= \lim_{x \to -2} \frac{\left(x + \sqrt{x+6}\right)\left(x - \sqrt{2-x}\right)}{x^2 + x - 2} = \lim_{x \to -2} \frac{\left(x + \sqrt{x+6}\right)\left(x - \sqrt{x+6}\right)\left(x - \sqrt{2-x}\right)}{\left(x^2 + x - 2\right)\left(x - \sqrt{x+6}\right)}$$

$$= \lim_{x\to -2} \frac{(x^2-x-6)(x-\sqrt{2-x})}{(x+2)(x-1)(x-\sqrt{x+6})} = \lim_{x\to -2} \frac{(x-3)\cancel{(x+2)}(x-\sqrt{2-x})}{\cancel{(x+2)}(x-1)(x-\sqrt{x+6})}$$

$$= \lim_{x\to -2} \frac{(x-3)(x-\sqrt{2-x})}{(x-1)(x-\sqrt{x+6})} = \frac{20}{12} = \frac{5}{3}$$

2.4.5 Calculer, si elles existent, la limite à gauche, la limite à droite et la limite des fonctions suivantes pour x tendant vers x_0 :

a) $f(x) = \dfrac{|x|}{x}$ $\quad x_0 = 0$

b) $f(x) = \dfrac{x^2+|x|}{|x|}$ $\quad x_0 = 0$

c) $f(x) = \dfrac{x^2-2x}{|x|}$ $\quad x_0 = 0$

d) $f(x) = \dfrac{|x-2|}{x^2-3x+2}$ $\quad x_0 = 2$

Rappel : $\forall x \in \mathbb{R}$

$$|x| = \begin{cases} x & \text{si } x>0 \\ 0 & \text{si } x=0 \\ -x & \text{si } x<0 \end{cases}$$

a) $$\lim_{\substack{x\to 0\\ <}} \frac{|x|}{x} = \lim_{\substack{x\to 0\\ <}} \frac{-x}{x} = -1$$

$$\lim_{\substack{x\to 0\\ >}} \frac{|x|}{x} = \lim_{\substack{x\to 0\\ >}} \frac{x}{x} = 1$$

$$\Rightarrow \lim_{\substack{x\to 0\\ <}} \frac{|x|}{x} \neq \lim_{\substack{x\to 0\\ >}} \frac{|x|}{x} \Rightarrow \lim_{x\to 0} \frac{|x|}{x} \text{ n'existe pas}$$

b) $$\lim_{\substack{x\to 0\\ <}} \frac{x^2+|x|}{|x|} = \lim_{\substack{x\to 0\\ <}} \frac{x^2-x}{-x} = \lim_{\substack{x\to 0\\ <}} \frac{x(x-1)}{-x} = \lim_{\substack{x\to 0\\ <}} -(x-1) = 1$$

$$\lim_{\substack{x\to 0\\ >}} \frac{x^2+|x|}{|x|} = \lim_{\substack{x\to 0\\ >}} \frac{x^2+x}{x} = \lim_{\substack{x\to 0\\ >}} \frac{x(x+1)}{x} = \lim_{\substack{x\to 0\\ >}} (x+1) = 1$$

$$\Rightarrow \lim_{\substack{x\to 0\\ <}} \frac{x^2+|x|}{|x|} = \lim_{\substack{x\to 0\\ >}} \frac{x^2+|x|}{|x|} = \lim_{x\to 0} \frac{x^2+|x|}{|x|} = 1$$

2.4.14 Calculer les limites suivantes :

a) $\lim\limits_{x\to-2} \dfrac{x^2+3x+6}{(x+2)^2}$

b) $\lim\limits_{x\to-3} \dfrac{x^2+2x-15}{x^2+8x+15}$

c) $\lim\limits_{x\to0} \dfrac{x^2-3x}{x^3}$

d) $\lim\limits_{\substack{x\to5\\>}} \dfrac{x-3}{5-x}$

e) $\lim\limits_{x\to1} (2x^2-5x+3)\dfrac{1}{x-1}$

f) $\lim\limits_{x\to1} \left(\dfrac{x^2}{x-1}-\dfrac{1}{x-1}\right)$

g) $\lim\limits_{\substack{x\to-2\\<}} \dfrac{x-1}{x+2}$

h) $\lim\limits_{x\to2} \left(\dfrac{1}{x-2}-\dfrac{4}{x^2-4}\right)$

a) $\lim\limits_{x\to-2} \dfrac{x^2+3x+6}{(x+2)^2} \overset{\frac{4}{0_+}}{=} \underline{+\infty}$

$x\to-2 \to (x+2)^2 \to 0_+$

b) $\lim\limits_{x\to-3} \dfrac{x^2+2x-15}{x^2+8x+15} = \lim\limits_{x\to-3} \dfrac{(\cancel{x+5})(x-3)}{(\cancel{x+5})(x+3)} = \lim\limits_{x\to-3} \dfrac{x-3}{x+3}$

* $\lim\limits_{\substack{x\to-3\\<}} \dfrac{x-3}{x+3} \overset{\frac{-6}{0_-}}{=} \underline{+\infty}$

* $\lim\limits_{\substack{x\to-3\\>}} \dfrac{x-3}{x+3} \overset{\frac{-6}{0_+}}{=} \underline{-\infty}$

c) $\lim\limits_{x\to0} \dfrac{x^2-3x}{x^3} \overset{\text{“}\frac{0}{0}\text{” Ii}}{=} \lim\limits_{x\to0} \dfrac{\cancel{x}(x-3)}{x^{\cancel{3}2}} = \lim\limits_{x\to0} \dfrac{x-3}{x^2} \overset{\frac{-3}{0_+}}{=} \underline{-\infty}$

d) $\lim\limits_{\substack{x\to5\\>}} \dfrac{x-3}{5-x} \overset{\frac{2}{0_-}}{=} \underline{-\infty}$

e) $\lim\limits_{x\to1} (2x^2-5x+3)\cdot\dfrac{1}{x-1} \overset{\text{“}\frac{0}{0}\text{” Ii}}{=} \lim\limits_{x\to1} \dfrac{(2x-3)(\cancel{x-1})}{\cancel{x-1}}$

$= \lim\limits_{x\to1} (2x-3) = \underline{-1}$

f) $\lim\limits_{x \to 1} \left(\frac{x^2}{x-1} - \frac{1}{x-1} \right) \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x \to 1} \left(\frac{x^2-1}{x-1} \right) = \lim\limits_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$= \lim\limits_{x \to 1} (x+1) = \underline{2}$

g) $\lim\limits_{\substack{x \to -2 \\ <}} \frac{x-1}{x+1} \overset{\frac{-3}{0^-}}{=} \underline{+\infty}$

h) $\lim\limits_{x \to 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right) = \lim\limits_{x \to 2} \left(\frac{1}{x-2} - \frac{4}{(x-2)(x+2)} \right)$

$= \lim\limits_{x \to 2} \frac{x+2-4}{(x-2)(x+2)} = \lim\limits_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} \overset{\text{"}\frac{0}{0}\text{" fi}}{=} \lim\limits_{x \to 2} \frac{1}{x+2} = \underline{\frac{1}{4}}$

2.4.16 Calculer les limites suivantes :

a) $\lim\limits_{x\to\infty} \frac{2x-4}{-3x+1}$

b) $\lim\limits_{x\to-\infty} \frac{-3x^2+1}{x+2}$

c) $\lim\limits_{x\to\infty} \frac{x^2+4x+29}{x^2-2x+4}$

d) $\lim\limits_{x\to\infty} \frac{(3x+4)(x-1)}{(2x+7)(1-5x)}$

e) $\lim\limits_{x\to\infty} \frac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8}$

f) $\lim\limits_{x\to\infty} \left(\frac{2x^2-1}{x-1}+1-2x\right)$

g) $\lim\limits_{x\to+\infty} \left(\frac{2x-x^3}{3x+1}+x-1\right)$

h) $\lim\limits_{x\to\infty} \left(\frac{1+5x-3x^2}{x-2}+3x+1\right)$

a) $\lim\limits_{x \to \infty} \frac{2x-4}{-3x+1} = \lim\limits_{x \to \infty} \frac{\cancel{x}\left(2 - \frac{4}{x}\right)}{\cancel{x}\left(-3 + \frac{1}{x}\right)} = \lim\limits_{x \to \infty} \frac{2 - \frac{4}{x}}{-3 + \frac{1}{x}} = \underline{-\frac{2}{3}}$

$\frac{4}{x} \to 0$, $\frac{1}{x} \to 0$

b) $\lim\limits_{x \to -\infty} \frac{-3x^2+1}{x+2} = \lim\limits_{x \to -\infty} \frac{\cancel{x}\left(-3x + \frac{1}{x}\right)}{\cancel{x}\left(1 + \frac{2}{x}\right)} = \lim\limits_{x \to -\infty} \frac{-3x + \frac{1}{x}}{1 + \frac{2}{x}}$

$\frac{1}{x} \to 0$, $\frac{2}{x} \to 0$

$= \lim\limits_{x \to -\infty} (-3x) = -3(-\infty) = \underline{+\infty}$

c) $\lim\limits_{x\to\infty} \dfrac{x^2+4x+29}{x^2-2x+4} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}\left(1+\underbrace{\frac{4}{x}}_{\to 0}+\underbrace{\frac{29}{x^2}}_{\to 0}\right)}{\cancel{x^2}\left(1-\frac{2}{x}+\underbrace{\frac{4}{x^2}}_{\to 0}\right)} = \lim\limits_{x\to\infty} 1 = \underline{1}$

d) $\lim\limits_{x\to\infty} \dfrac{(3x+4)(x-1)}{(2x+7)(1-5x)} = \lim\limits_{x\to\infty} \dfrac{\cancel{x}\left(3+\underbrace{\frac{4}{x}}_{\to 0}\right)\cdot x\left(1-\underbrace{\frac{1}{x}}_{\to 0}\right)}{\cancel{x}\left(2+\underbrace{\frac{7}{x}}_{\to 0}\right)\cdot x\left(\underbrace{\frac{1}{x}}_{\to 0}-5\right)}$

$= \lim\limits_{x\to\infty} \dfrac{\left(3+\frac{4}{x}\right)\left(1-\frac{1}{x}\right)}{\left(2+\frac{7}{x}\right)\left(\frac{1}{x}-5\right)} = \lim\limits_{x\to\infty} \dfrac{3}{(2)(-5)} = \underline{-\dfrac{3}{10}}$

Rf e) $\lim\limits_{x\to\infty} \dfrac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8} = \lim\limits_{x\to\infty} \dfrac{\left(1+\frac{1}{x}\right)^7\left(2+\frac{3}{x}\right)^4}{\left(2+\frac{1}{x}\right)^3\left(1-\frac{98}{x}\right)^8} = \underline{2}$

f) $\lim\limits_{x\to\infty} \left(\dfrac{2x^2-1}{x-1}+1-2x\right) = \lim\limits_{x\to\infty} \left(\dfrac{2x^2-1+(1-2x)(x-1)}{x-1}\right)$

$= \lim\limits_{x\to\infty} \left(\dfrac{2x^2-1+x-1-2x^2+2x}{x-1}\right) = \lim\limits_{x\to\infty} \left(\dfrac{3x-2}{x-1}\right)$

$= \lim\limits_{x\to\infty} \dfrac{\cancel{x}\left(3-\frac{2}{x}\right)}{\cancel{x}\left(1-\frac{1}{x}\right)} = \lim\limits_{x\to\infty} \dfrac{3-\frac{2}{x}}{1-\frac{1}{x}} = \underline{3}$

Rf g) $\lim\limits_{x\to+\infty} \left(\dfrac{2x-x^3}{3x+1}+x-1\right) = \lim\limits_{x\to+\infty} \left(\dfrac{2x-x^3+(x-1)(3x+1)}{3x+1}\right)$

$= \lim\limits_{x\to+\infty} \left(\dfrac{2x-x^3+3x^2+x-3x-1}{3x+1}\right) = \lim\limits_{x\to+\infty} \left(\dfrac{-x^3+3x^2-1}{3x+1}\right)$

$= \lim\limits_{x\to+\infty} \dfrac{\cancel{x}\left(-x^2+3x-\frac{1}{x}\right)}{\cancel{x}\left(3+\frac{1}{x}\right)} = \lim\limits_{x\to+\infty} \dfrac{-x^2+3x-\frac{1}{x}}{3+\frac{1}{x}}$

$= \lim\limits_{x\to+\infty} \dfrac{-x^2+3x}{3} = \lim\limits_{x\to+\infty} \dfrac{\cancel{x^2}\left(-1+\frac{3}{x}\right)}{\cancel{x^2}\,\frac{3}{x^2}} = \lim\limits_{x\to+\infty} \dfrac{-1+\frac{3}{x}}{\frac{3}{x^2}}$

$= \underline{-\infty}$

h) $\lim_{x \to \infty} \left(\frac{1+5x-3x^2}{x-2} + 3x + 1 \right) = \lim_{x \to \infty} \left(\frac{1+5x-3x^2+(3x+1)(x-2)}{x-2} \right)$

$= \lim_{x \to \infty} \left(\frac{1+\cancel{5x}-\cancel{3x^2}+\cancel{3x^2}-\cancel{6x}+\cancel{x}-2}{x-2} \right) = \lim_{x \to \infty} \left(\frac{-1}{x-2} \right) = \underline{0}$