

Asymptotes et limites

1) Soit la fonction définie par :

$$f(x) = \sqrt{4x^2 + 4x - 3} - 2x + 6$$

a) Déterminer ED_f .

b) Déterminer le type de $f(x)$.

c) Déterminer toutes les asymptotes de la courbe $y = f(x)$

2) Rechercher toutes les asymptotes à la courbe représentative de la fonction

$$f(x) = \frac{-2x^3 - x^2 + 5x + 1}{x^4 + x - 2}$$

3) Soit la fonction définie par

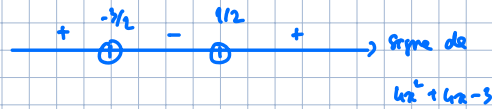
$$f(x) = \frac{3x^3 + 7x^2 + 5x - 2}{x^2 + 2x}$$

Déterminer la position de la courbe $y = f(x)$ par rapport à son asymptote oblique.

4) $\lim_{x \rightarrow \infty} \left(x \cdot \sin \left(\frac{2x}{x^2 + 1} \right) \right)$

$$1) f(x) = \sqrt{4x^2 + 4x - 3} - 2x + 6$$

a) condição: $4x^2 + 4x - 3 \geq 0 \quad (\Rightarrow) \quad (2x+3)(2x-1) \geq 0$



$$\Rightarrow \text{E.O.} = \left] -\infty; -\frac{3}{2} \right] \cup \left[\frac{1}{2}; +\infty \right[$$

b) Signo de f(x):

+ zero de f(x): $\sqrt{4x^2 + 4x - 3} - 2x + 6 = 0$

$$\Rightarrow \sqrt{4x^2 + 4x - 3} = 2x - 6 \quad | \wedge^2$$

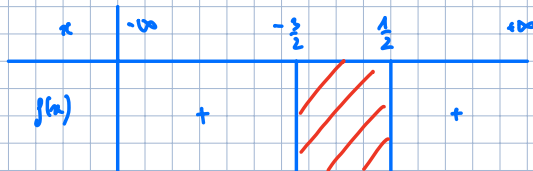
condição $2x - 6 \geq 0$
 $x \geq 3$

$$\left(\sqrt{4x^2 + 4x - 3} \right)^2 = (2x - 6)^2$$

$$\Rightarrow 4x^2 + 4x - 3 = 4x^2 - 24x + 36$$

$$\Rightarrow 2 = \frac{39}{28} \quad \text{ne convém pros}$$

\(\Rightarrow\) f(x) não tem zero



c) + AV: nenhuma

+ áspicte:

$$\bullet m = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 4x - 3} - 2x + 6}{x}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 \left(1 + \frac{1}{x} - \frac{3}{4x}\right)} - 2x + 6}{x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x \sqrt{1 + \frac{1}{x} - \frac{3}{4x}} - 2x + 6}{x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x \left(\sqrt{1 + \frac{1}{x} - \frac{3}{4x}} + 1 - \frac{3}{2x} \right)}{x}$$

$$\Rightarrow m = \lim_{x \rightarrow -\infty} -2(1+1) = -4$$

$$\begin{aligned} \bullet h &= \lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 + 4x - 3} - 2x + 6 + 4x \right) \\ &= \lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 + 4x - 3} + 2x + 6 \right) \end{aligned}$$

$$\stackrel{-\infty + \infty}{=} \lim_{x \rightarrow -\infty} \frac{\left(\sqrt{4x^2 + 4x - 3} + 2x + 6 \right) \left(\sqrt{4x^2 + 4x - 3} - (2x + 6) \right)}{\sqrt{4x^2 + 4x - 3} - (2x + 6)}$$

$$= \lim_{x \rightarrow -\infty} \frac{4x^2 + 4x - 6 - (2x + 6)^2}{-2x \sqrt{1 + \frac{1}{2} - \frac{3}{2x}} - 2x \left(1 + \frac{3}{x} \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{4x^2 + 4x - 6 - 4x^2 - 24x - 36}{-2x \left(\sqrt{1 + \frac{1}{2} - \frac{3}{2x}} + 1 + \frac{3}{x} \right)} = \lim_{x \rightarrow -\infty} \frac{-20x - 39}{-2x \left(\sqrt{1 + \dots} + 1 + \dots \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x \left(20 + \frac{39}{x} \right)}{-2x \left(\sqrt{1 + \dots} + 1 + \dots \right)} = \frac{20}{2(1+1)} = \frac{20}{4} = 5$$

\(\Rightarrow\) \(\hat{A}\) gauche il y a une AD : $y = -4x + 5$

* \(\hat{A}\) droite :

$$\lim_{x \rightarrow +\infty} \left(\sqrt{4x^2 + 4x - 3} - 2x + 6 \right) = \lim_{x \rightarrow +\infty} \frac{\left(\sqrt{4x^2 + 4x - 3} - (2x - 6) \right) \left(\sqrt{4x^2 + 4x - 3} + (2x - 6) \right)}{\sqrt{4x^2 + 4x - 3} + 2x - 6}$$

$$= \lim_{x \rightarrow +\infty} \frac{4x^2 + 4x - 3 - 4x^2 + 24x - 36}{\sqrt{4x^2 + 4x - 3} + 2x - 6} = \lim_{x \rightarrow +\infty} \frac{28x - 39}{2x \left(\sqrt{1 + \dots} + 2x(1 - \dots) \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(28 - \frac{39}{x} \right)}{2x \left(\sqrt{1 + \dots} + 1 - \dots \right)} = \frac{28}{2 \cdot 2} = 7$$

\(\Rightarrow\) \(\hat{A}\) droite il y a une AD : $y = 7$

$$2) \quad f(x) = \frac{-2x^3 - x^2 + 5x + 1}{x^2 + x - 2} = \frac{-2x^3 - x^2 + 5x + 1}{(x+2)(x-1)}$$

$$\Rightarrow \text{ED} = \mathbb{R} \setminus \{-2; 1\}$$

* AV:

$$\textcircled{a} \quad \lim_{x \rightarrow -2} \frac{-2x^3 - x^2 + 5x + 1}{(x+2)(x-1)} \stackrel{\frac{0}{0}}{=} \infty \Rightarrow \text{AV : } x = -2$$

$$\textcircled{b} \quad \lim_{x \rightarrow 1} \frac{-2x^3 - x^2 + 5x + 1}{(x+2)(x-1)} \stackrel{\frac{0}{0}}{=} \infty \Rightarrow \text{AV : } x = 1$$

* AD:

$$\begin{array}{r|l} -2x^3 - x^2 + 5x + 1 & x^2 + x - 2 \\ - (-2x^3 - 2x^2 + 4x) & -2x + 1 \\ \hline 0 & x^2 + x + 1 \\ - & x^2 + x - 2 \\ \hline & 3 \end{array}$$

$$\Rightarrow \text{AV : } y = -2x + 1$$

$$3) \quad f(x) = \frac{3x^3 + 7x^2 + 3x - 2}{x^2 + 2x}$$

on cherche AO:

$$\begin{array}{r|l} 3x^3 + 7x^2 + 3x - 2 & x^2 + 2x \\ - 3x^3 + 6x^2 & \underline{3x + 1} \\ \hline x^2 + 3x & \downarrow \\ - x^2 + 2x & \text{AO} \\ \hline x - 2 & \end{array}$$

$$\Rightarrow f(x) = 3x + 1 + \frac{x-2}{x^2+2x}$$

$$\text{d'où } g(x) = \frac{x-2}{x^2+2x} = \frac{x-2}{x(x+2)}$$

$$\text{ED}_{g(x)} = \mathbb{R}^* \setminus \{-2\}$$

x	$-\infty$	-2	0	2	$+\infty$
g(x)	-	+	-	○	+
Position	au dessus	au dessous	au dessus	○	au dessus

↑
intersection

$$h) \lim_{x \rightarrow \infty} \left(x \cdot \ln \left(\frac{2x}{x^2+1} \right) \right)$$

$$\text{On pose } t = \frac{1}{x} \quad \Rightarrow \quad x = \frac{1}{t}$$

Quand $x \rightarrow \infty$ alors $t \rightarrow 0$

$$\text{Donc } \lim_{\substack{t \rightarrow 0 \\ <}} \left(\frac{1}{t} \cdot \ln \left(\frac{\frac{2}{t}}{\frac{1}{t^2}+1} \right) \right) = \lim_{\substack{t \rightarrow 0 \\ <}} \frac{\ln \left(\frac{\frac{2}{t}}{\frac{1+t^2}{t^2}} \right)}{t}$$

$$= \lim_{\substack{t \rightarrow 0 \\ <}} \frac{\ln \left(\frac{2t}{1+t^2} \right)}{t} = \lim_{\substack{t \rightarrow 0 \\ <}} \frac{2}{t^2+1} \cdot \frac{\ln \left(\frac{2t}{1+t^2} \right)}{\frac{2t}{1+t^2}}$$

$$= \lim_{\substack{t \rightarrow 0 \\ <}} \frac{2}{t^2+1} \cdot \lim_{\substack{t \rightarrow 0 \\ <}} \frac{\ln \left(\frac{2t}{1+t^2} \right)}{\frac{2t}{1+t^2}} = 2 \cdot 1 = 2$$

$\underbrace{\hspace{10em}}_{\rightarrow 1}$

De même, lorsque $x \rightarrow +\infty \Rightarrow \lim_{x \rightarrow \infty} \left(x \cdot \ln \left(\frac{2x}{x^2+1} \right) \right) = 2$