CORRIGÉ

2.5 Equations et systèmes d'équations

2.5.1 Résoudre les équations ci-dessous :

a) $4(x-3)+x(x-5)-30=0$

b) $(x+1)(x+2)+(x+3)(x+4)=42$

c) $(x-2)(x-4)+(x+3)(x-1)=39$

a) $4(x-3)+x(x-5)-30=0$

$\Leftrightarrow 4x-12+x^2-5x-30=0 \Leftrightarrow x^2-x-42=0 \Leftrightarrow (x-7)(x+6)=0$

$\Leftrightarrow \boxed{S=\{-6;7\}}$

b) $(x+1)(x+2)+(x+3)(x+4)=42$

$\Leftrightarrow x^2+2x+x+2+x^2+4x+3x+12=42 \Leftrightarrow x^2+5x-14=0$

$\Leftrightarrow (x+7)(x-2)=0 \Leftrightarrow \boxed{S=\{-7;2\}}$

c) $(x-2)(x-4)+(x+3)(x-1)=39$

$\Leftrightarrow x^2-6x+8+x^2+2x-3=39 \Leftrightarrow x^2-2x-17=$

$\Leftrightarrow \Delta=(-2)^2-4\cdot 1\cdot(-17)=4+68=72=6\sqrt{2}$

$\Leftrightarrow x_1=\frac{2-6\sqrt{2}}{2}=\frac{2(1-3\sqrt{2})}{2}=1-3\sqrt{2}$

$x_2=\frac{2+6\sqrt{2}}{2}=\frac{2(1+3\sqrt{2})}{2}=1+3\sqrt{2}$

$\Leftrightarrow \boxed{S=\{1-3\sqrt{2};\ 1+3\sqrt{2}\}}$

d) $(x-6)(x+1)+(2x+3)(x-5)=0$

e) $(3x-5)^2-12x=1$

f) $x-7=6-(x-7)^2$

g) $(5x-1)^2+x^2+3=0$

h) $2(3x+1)^2-32(3x+1)+126=0$

i) $(x+1)^2-(x-1)^2=(x-8)^2$

j) $(4x+1)^2-(3x+1)^2=(2x+1)^2$

k) $(x+3)^3-(x-4)^3=721$

l) $(x-5)^3-(x+2)^3+91=0$

m) $(2x+1)^2-(x-1)(x+11)=(3x-2)^2-(3x-4)^2$

n) $(x+5)^2-(2x-1)(3x+5)=(x+3)^2-(x+1)^2$

o) $(4x-3)(2x-1)-(3x+5)^2=(2x+3)^2-(4x-1)(2x+7)-73$

d) $(x-6)(x+1)+(2x+3)(x-5)=0$

$\Leftrightarrow x^2+x-6x-6+2x^2-10x+3x-15=0 \Leftrightarrow x^2-4x-7=0$

$\Rightarrow \Delta = 44 = 2\sqrt{11}$

$\Rightarrow x_1 = \frac{4-2\sqrt{11}}{2} = \frac{2(2-\sqrt{11})}{2} = 2-\sqrt{11}$

$x_2 = \frac{4+2\sqrt{11}}{2} = \frac{2(2+\sqrt{11})}{2} = 2+\sqrt{11}$

$\Rightarrow S = \{2-\sqrt{11}\ ;\ 2+\sqrt{11}\}$

e) $(3x-5)^2 - 12x = 1$

$\Leftrightarrow 9x^2 - 30x + 25 - 12x = 1 \Leftrightarrow 3x^2 - 14x + 8 = 0$

$\Leftrightarrow (3x-2)(x-4) = 0 \Leftrightarrow S = \left\{ \frac{2}{3} ; 4 \right\}$

f) $x - 7 = 6 - (x-7)^2$

$\Leftrightarrow x - 7 = 6 - (x^2 - 14x + 49) = 6 - x^2 + 14x - 49$

$\Leftrightarrow x^2 - 13x + 36 = 0 \Leftrightarrow (x-4)(x-9) = 0$

$\Leftrightarrow S = \{ 4 ; 9 \}$

g) $(5x-1)^2 + x^2 + 3 = 0$

$\Leftrightarrow 25x^2 - 10x + 1 + x^2 + 3 = 0 \Leftrightarrow 26x^2 - 10x + 4 = 0$

$\Leftrightarrow \Delta = -316 < 0 \Leftrightarrow S = \emptyset$

h) $2(3x+1)^2 - 32(3x+1) + 126 = 0$

$\Leftrightarrow 18x^2 + 12x + 2 - 96x - 32 + 126 = 0 \Leftrightarrow 3x^2 - 14x + 16 = 0$

$\Leftrightarrow (3x-8)(x-2) = 0 \Leftrightarrow S = \left\{ 2 ; \frac{8}{3} \right\}$

i) $(x+1)^2 - (x-1)^2 = (x-8)^2$

$\Leftrightarrow x^2 + 2x + 1 - x^2 + 2x - 1 = x^2 - 16x + 64 \Leftrightarrow x^2 - 20x + 64 = 0$

$\Leftrightarrow (x-4)(x-16) = 0 \Leftrightarrow S = \{ 4 ; 16 \}$

2.5.2 Résoudre l'équation $2x^2+7x-15=0$. Puis factoriser le polynôme $2x^2+7x-15$. Factoriser les polynômes ci-dessous d'une manière analogue.

a) $2x^2-7x-4$

b) $6x^2+11x+4$

c) $6x^2-25x-25$

d) $6x^2-20x+25$

e) $12x^2+23x-24$

f) $5x^2+\frac{29}{3}x-\frac{14}{3}$

$2x^2+7x-15=0 \Rightarrow \Delta = 169 \Rightarrow x_1 = \frac{3}{2}, x_2 = -5$

$\Rightarrow 2x^2+7x-15 = 2\left(x-\frac{3}{2}\right)(x+5)$

$\Rightarrow 2x^2+7x-15 = \cancel{2}\left(\frac{2x-3}{\cancel{2}}\right)(x+5) = \boxed{(2x-3)(x+5)}$

a) $2x^2-7x-4=0$

$\Delta = 81 \Rightarrow 2x^2-7x-4 = \boxed{(x-4)(2x+1)}$

b) $6x^2+11x+4=0$

$\Delta = 25 \Rightarrow 6x^2+11x+4 = \boxed{(2x+1)(3x+4)}$

c) $6x^2-25x-25=0$

$\Delta = (-25)^2-4\cdot 6(-25) = 625+600 = 1225 \Rightarrow \sqrt{\Delta} = 35$

$\Rightarrow 6x^2-25x-25 = \boxed{(x-5)(6x+5)}$

d) $6x^2-20x+25=0$

$\Rightarrow \Delta = (-20)^2-4\cdot 6\cdot 25 = 400-600 = -200 < 0$

$\Rightarrow$ pas factorisable

e) $12x^2 + 23x - 24 = 0$

$\Delta = (23)^2 - 4 \cdot 12 \cdot (-24) = 529 + 1152 = 1681 = 41^2$

$\Rightarrow 12x^2 + 23x - 24 = \boxed{(3x + 8)(4x - 3)}$

f) $5x^2 + \frac{29}{3}x - \frac{14}{3} = 0$

$\Delta = \left(\frac{29}{3}\right)^2 - 4 \cdot 5 \left(-\frac{14}{3}\right) = \frac{841}{9} + \frac{840}{9} = \frac{1681}{9} = \frac{41^2}{3^2}$

$x_1 = \dfrac{-\frac{29}{3} - \frac{41}{3}}{10} = \dfrac{-\frac{70}{3}}{10} = -\frac{70}{30} = -\frac{7}{3}$

$x_2 = \dfrac{-\frac{29}{3} + \frac{41}{3}}{10} = \frac{4}{10} = \frac{2}{5}$

$\Rightarrow 5x^2 + \frac{29}{3}x - \frac{14}{3} = 5\left(x + \frac{7}{3}\right)\left(x - \frac{2}{5}\right)$

$= \cancel{5}\left(x + \frac{7}{3}\right)\left(\frac{5x - 2}{\cancel{5}}\right)$

$5x^2 + \frac{29}{3}x - \frac{14}{3} = \boxed{(5x - 2)\left(x + \frac{7}{3}\right)}$

2.5.3 Résoudre les équations.

a) $x^2 - 9 = 0$

b) $4x^2 - 1 = 0$

c) $(x-2)^2 - 9(x-2) = 0$

d) $(x^2 - x - 6)(x+5) = 0$

e) $x^4 - 5x^2 + 4 = 0$

f) $(x-1)(x^2+1) = 0$

g) $x^3 + x^2 = 4x + 4$

h) $x^2 - 9 - 4(x-3) = 0$

i) $(x+6)^2 - 3(x+6) + 2 = 0$

j) $x^3 + 2x^2 - x - 2 = 0$

a) $x^2 - 9 = 0 \Leftrightarrow (x-3)(x+3) = 0 \Rightarrow S = \{-3 ; 3\}$

b) $4x^2 - 1 = 0 \Leftrightarrow (2x)^2 - 1 = 0 \Leftrightarrow (2x-1)(2x+1) = 0$

$\Rightarrow S = \left\{-\frac{1}{2} ; \frac{1}{2}\right\}$

c) $(x-2)^2 - 9(x-2) = 0$

$\Leftrightarrow (x-2)(x-2-9) = 0 \Leftrightarrow (x-2)(x-11) = 0$

$\Rightarrow S = \{2 ; 11\}$

d) $(x^2 - x - 6)(x+5) = 0$

$\Downarrow$

$(x-3)(x+2)(x+5) = 0 \Rightarrow S = \{-5 ; -2 ; 3\}$

e) $x^4 - 5x^2 + 4 = 0$

On pose $y = x^2$

$\Rightarrow y^2 - 5y + 4 = 0 \Leftrightarrow (y-1)(y-4) = 0$

$\Rightarrow (x^2-1)(x^2-4) = 0 \Leftrightarrow (x-1)(x+1)(x-2)(x+2) \Rightarrow S = \{-2 ; -1 ; 1 ; 2\}$

f) $(x-1)(x^2+1) = 0 \iff S = \{1\}$

x^2+1 : pas factorisable

g) $x^3 + x^2 = 4x + 4$

$\iff x^3 + x^2 - 4x - 4 = 0 \iff x^2(x+1) - 4(x+1) = 0$

$\iff (x+1)(x^2-4) = 0 \iff (x+1)(x-2)(x+2) = 0$

$\iff S = \{-2 ; -1 ; 2\}$

h) $x^2 - 9 - 4(x-3) = 0$

$\iff (x-3)(x+3) - 4(x-3) = 0 \iff (x-3)(x+3-4) = 0$

$\iff (x-3)(x-1) = 0$

$\iff x_1 = 3 \quad , x_2 = 1$

$\iff S = \{1 ; 3\}$

i) $(x+6)^2 - 3(x+6) + 2 = 0$

On pose $x+6 = y \iff y^2 - 3y + 2 = 0$

$\iff (y-1)(y-2) = 0 \iff (x+6-1)(x+6-2) = 0$

$\iff (x+5)(x+4) \iff S = \{-5 ; -4\}$

j) $x^3 + 2x^2 - x - 2 = 0$

$\iff x^2(x+2) - (x+2) = 0 \iff (x+2)(x^2-1) = 0$

$\iff (x+2)(x-1)(x+1) = 0 \iff S = \{-2 ; -1 ; 1\}$

2.5.4 Résoudre les équations.

a) $(x^2 - 8x + 12)(x+2)^3 = 0$

b) $(x-3)(x^2-4) = 0$

c) $x^3 + 2x^2 - 4x = 8$

d) $(2x^2+3x+1)^2 - (2x^2-4x-1)^2 = 0$

e) $x(x-2) + (x-3)(x-2) = 0$

f) $6x^2 = 3x^3 - 72x$

g) $x^3 + 3x^2 = 9x + 27$

h) $(x-1)(x-2)(x-3) = x(x^2-9)$

a) $(x^2 - 8x + 12)(x+2)^3 = 0$

$(x-6)(x-2)(x+2)^3 = 0$

$\Leftrightarrow$ $x_1 = 6$

$x_2 = 2$

$x_3 = -2$

$\Leftrightarrow S = \{-2 ; 2 ; 6\}$

b) $(x-3)(x^2-4) = 0$

$\Leftrightarrow (x-3)(x-2)(x+2) = 0$ $\Leftrightarrow S = \{-2 ; 2 ; 3\}$

c) $x^3 + 2x^2 - 4x = 8$

$\Leftrightarrow x^3 + 2x^2 - 4x - 8 = 0$ $\Leftrightarrow x^2(x+2) - 4(x+2) = 0$

$\Leftrightarrow (x+2)(x^2-4) = 0$ $\Leftrightarrow (x+2)(x-2)(x+2) = (x+2)^2(x-2) = 0$

$\Leftrightarrow S = \{-2 ; 2\}$

d) $(2x^2+3x+1)^2 - (2x^2-4x-1)^2 = 0$

$\Leftrightarrow (2x^2+3x+1-(2x^2-4x-1))(2x^2+3x+1+2x^2-4x-1) = 0$

$\Leftrightarrow (2x^2+3x+1-2x^2+4x+1)(4x^2-x) = 0$

$\Leftrightarrow (7x+2)(4x^2-x) = 0 \Leftrightarrow (7x+2)x(4x-1) = 0$

$\Rightarrow x_1 = -\frac{2}{7}$

$x_2 = 0$

$x_3 = \frac{1}{4}$

$\Rightarrow \boxed{S = \left\{-\frac{2}{7}\ ;\ 0\ ;\ \frac{1}{4}\right\}}$

e) $x(x-2) + (x-3)(x-2) = 0$

$\Rightarrow (x-2)(x+x-3) = 0 \Leftrightarrow (x-2)(2x-3) = 0$

$\Rightarrow \boxed{S = \left\{\frac{3}{2}\ ;\ 2\right\}}$

f) $6x^2 = 3x^3 - 72x$

$\Leftrightarrow 3x^3 - 6x^2 - 72x = 0$

$\Leftrightarrow 3x(x^2 - 2x - 24) = 0$

$\Leftrightarrow 3x(x-6)(x+4) = 0 \Rightarrow \boxed{S = \{-4\ ;\ 0\ ;\ 6\}}$

g) $x^3 + 3x^2 = 9x + 27$

$\Leftrightarrow x^3 + 3x^2 - 9x - 27 = 0 \Leftrightarrow x^2(x+3) - 9(x+3) = 0$

$\Leftrightarrow (x+3)(x^2-9) = 0 \Leftrightarrow (x+3)(x-3)(x+3) = 0 \Leftrightarrow (x+3)^2(x-3) = 0$

$\Rightarrow \boxed{S = \{-3\ ;\ 3\}}$

h) $(x-1)(x-2)(x-3) = x(x^2-9)$

$\Leftrightarrow (x-1)(x-2)(x-3) = x(x-3)(x+3)$

$\Leftrightarrow (x-1)(x-2)(x-3) - x(x-3)(x+3) = 0 \Leftrightarrow (x-3)\left[(x-1)(x-2) - x(x+3)\right] = 0$

$\Leftrightarrow (x-3)(x^2 - 3x + 2 - x^2 - 3x) = 0 \Leftrightarrow (x-3)(-6x+2) = 0$

$\Leftrightarrow \boxed{S = \left\{\frac{1}{3}\,;\ 3\right\}}$

2.5.5 Résoudre l'équation $(x^2-2)^2 - 5(x^2-2) - 14 = 0$ (*)

poisons $x^2 - 2 = y$

(*) $\Leftrightarrow y^2 - 5y - 14 = 0$

(trinôme unitaire) $\rightarrow (y-7)(y+2) = 0 \Leftrightarrow (x^2-2-7)(x^2-2+2) = 0$

$\Leftrightarrow (x^2-9)(x^2) = 0 \Leftrightarrow x^2(x-3)(x+3) = 0$

$\Leftrightarrow x_1 = -3,\ x_2 = 0,\ x_3 = 3$

$\Leftrightarrow \boxed{S = \{-3\,;\ 0\,;\ 3\}}$

2.5.6 Résoudre les équations suivantes.

a) $x^4 - 13x^2 + 36 = 0$

b) $x^4 - 1 = 0$

c) $x^4 + 2x^2 + 1 = 0$

d) $x^6 - 7x^3 - 8 = 0$

e) $(x^2 - 1)^2 - 2(x^2 - 1) + 1 = 0$

f) $\frac{1}{x^2} - \frac{2}{x} - 3 = 0$

g) $(x^2 - 5x + 6)^2 - 2(x^2 - 5x + 6) = 0$

a) $x^4 - 13x^2 + 36 = 0$

posons $x^2 = y$

$\Leftrightarrow y^2 - 13y + 36 = 0 \Leftrightarrow (y-9)(y-4) = 0$

↑ trinôme unitaire

$\Leftrightarrow (x^2-9)(x^2-4) = 0$

$\Leftrightarrow (x-3)(x+3)(x-2)(x+2) = 0$

$\Leftrightarrow x_1 = -3, x_2 = -2, x_3 = 2, x_4 = 3$

$\Leftrightarrow \boxed{S = \{-3; -2; 2; 3\}}$

b) $x^4 - 1 = 0$

$\Leftrightarrow (x^2-1)(x^2+1) = 0 \Leftrightarrow (x-1)(x+1)(x^2+1) = 0$

x^2+1 par factorisable

$\Leftrightarrow x_1 = -1, x_2 = 1$

$\Leftrightarrow \boxed{S = \{-1; 1\}}$

c) $x^4 + 2x^2 + 1 = 0$

$\Leftrightarrow (x^2+1)^2 = 0$

x^2+1 par factorisable

$\Leftrightarrow$ pas de solution.

d) $x^6 - 7x^3 - 8 = 0$

poser $x^3 = y$ $\Rightarrow$ $y^2 - 7y - 8 = 0$

$\Leftrightarrow (y-8)(y+1) = 0$

$\Leftrightarrow (x^3 - 8)(x^3 + 1) = 0$ — identités remarquables !

$\Leftrightarrow (x-2)(x^2 + 2x + 4)(x+1)(x^2 - x + 1) = 0$

$x^2 + 2x + 4$: pas factorisable ; $x^2 - x + 1$: pas factorisable

$\Rightarrow x_1 = -1$, $x_2 = 2$ $\Rightarrow$ $S = \{-1 ; 2\}$

g) $(x^2 - 5x + 6)^2 - 2(x^2 - 5x + 6) = 0$

$\Leftrightarrow (x^2 - 5x + 6)(x^2 - 5x + 6 - 2) = 0$

$\Leftrightarrow (x^2 - 5x + 6)(x^2 - 5x + 4) = 0$

$\Leftrightarrow (x-3)(x-2)(x-4)(x-1) = 0$ $\Rightarrow x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$

$\Rightarrow S = \{1 ; 2 ; 3 ; 4\}$

2.5.7 Résoudre les équations.

a) $\dfrac{x-8}{5}+\dfrac{x^2}{3}=1$

b) $\dfrac{3x-7}{5}+\dfrac{x^2-9}{7}=2$

c) $\dfrac{1-8x}{2}-\dfrac{x^2-7}{4}+2x=0$

d) $\dfrac{x^2-3}{2}-\dfrac{x^2+1}{3}=\dfrac{x^2-11}{6}$

e) $\dfrac{3x+1}{8}-\dfrac{x^2+5}{4}=\dfrac{55}{2}$

f) $\dfrac{(x-2)^2}{5}-\dfrac{(x-3)^2}{4}=0$

g) $\dfrac{2x^2}{3}+\dfrac{7}{2}=\dfrac{x}{2}+8$

h) $x=\dfrac{2}{5}+\dfrac{5x^2}{16}$

i) $\dfrac{x^2}{3}+\dfrac{4x}{5}-19=\dfrac{76}{5}$

j) $\dfrac{5-4x}{2}+\dfrac{3x^2-1}{3}=\dfrac{2x^2+5}{6}$

k) $\dfrac{x^2+5}{8}=\dfrac{2(3-x)}{5}-\dfrac{3(x-1)}{10}$

l) $\dfrac{x^2-10}{9}-\dfrac{3(4-x)}{4}=\dfrac{2(x-3)}{3}$

a) $\dfrac{x-8}{5}+\dfrac{x^2}{3}=1$

$\Leftrightarrow \quad 3(x-8)+5x^2=15 \quad \Leftrightarrow \quad 3x-24+5x^2-15=0$

$\Leftrightarrow \quad 5x^2+3x-39=0$

$\Delta = 789 \quad \Rightarrow \quad x_1=\dfrac{-3-\sqrt{789}}{10}, \quad x_2=\dfrac{-3+\sqrt{789}}{10}$

$\Rightarrow \quad S=\left\{\dfrac{-3-\sqrt{789}}{10}\,;\,\dfrac{-3+\sqrt{789}}{10}\right\}$

b) $\frac{3x-7}{5} + \frac{x^2-9}{7} = 2$

$\Leftrightarrow 7(3x-7) + 5(x^2-9) = 70$

$\Leftrightarrow 21x - 49 + 5x^2 - 45 - 70 = 0 \Leftrightarrow 5x^2 + 21x - 164 = 0$

$\Leftrightarrow \Delta = 3721 = 61^2 \Rightarrow x_1 = -\frac{41}{5}$; $x_2 = 4$

$\Rightarrow \boxed{S = \left\{ -\frac{41}{5} ; 4 \right\}}$

d) $\frac{x^2-3}{2} - \frac{x^2+1}{3} = \frac{x^2-11}{6}$

$3(x^2-3) - 2(x^2+1) = x^2 - 11 \Leftrightarrow 3x^2 - 9 - 2x^2 - 2 - x^2 + 11 = 0$

$\Leftrightarrow 0x = 0 \Rightarrow \boxed{S = \mathbb{R}}$

f) $\frac{(x-2)^2}{5} - \frac{(x-3)^2}{4} = 0$

$\Leftrightarrow 4(x-2)^2 - 5(x-3)^2 = 0$

$\Leftrightarrow 4(x^2 - 4x + 4) - 5(x^2 - 6x + 9) = 0$

$\Leftrightarrow 4x^2 - 16x + 16 - 5x^2 + 30x - 45 = 0 \Leftrightarrow -x^2 + 14x - 29 = 0$

$\Leftrightarrow x^2 - 14x + 29 = 0$

$\Delta = 80 \Rightarrow \sqrt{\Delta} = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$

$x_1 = \frac{14 - 4\sqrt{5}}{2} = \frac{2(7 - 2\sqrt{5})}{2} = 7 - 2\sqrt{5}$

$x_2 = \frac{14 + 4\sqrt{5}}{2} = \frac{2(7 + 2\sqrt{5})}{2} = 7 + 2\sqrt{5}$

$\Rightarrow \boxed{S = \left\{ 7 - 2\sqrt{5} ; 7 + 2\sqrt{5} \right\}}$

2.5.8 Résoudre les équations suivantes.

a) $\frac{1}{4}x + \frac{2}{5} = \frac{1}{5}x - \frac{3}{4}$

b) $3x + 8 = 2(x + 4)$

c) $2x + 5 = \frac{1}{2}(7 - 4x)$

d) $\frac{1}{2}(8 + 2x) = x + 4$

e) $\frac{t-5}{3} = \frac{2-t}{2}$

f) $3x - \frac{4-x}{2} = x - \frac{1}{3}$

a) $\frac{1}{4}x + \frac{2}{5} = \frac{1}{5}x - \frac{3}{4}$

$\Leftrightarrow 5x + 8 = 4x - 15 \Leftrightarrow x = -23 \Rightarrow S = \{-23\}$

b) $3x + 8 = 2(x+4)$

$\Leftrightarrow 3x + 8 = 2x + 8 \Leftrightarrow x = 0 \Rightarrow S = \{0\}$

c) $2x + 5 = \frac{1}{2}(7 - 4x)$

$\Leftrightarrow 2(2x + 5) = 7 - 4x \Leftrightarrow 4x + 10 = 7 - 4x$

$\Leftrightarrow 8x = -3 \Leftrightarrow x = -\frac{3}{8} \Rightarrow S = \left\{-\frac{3}{8}\right\}$

d) $\frac{1}{2}(8 + 2x) = x + 4$

$\Leftrightarrow 8 + 2x = 2x + 8 \Leftrightarrow 0 = 0 \Rightarrow S = \mathbb{R}$

e) $\frac{t-5}{3} = \frac{2-t}{2} \Leftrightarrow 2(t-5) = 3(2-t)$

$\Leftrightarrow 2t - 10 = 6 - 3t \Leftrightarrow 5t = 16 \Rightarrow t = \frac{16}{5} \Rightarrow S = \left\{\frac{16}{5}\right\}$

f) $3x - \frac{4-x}{2} = x - \frac{1}{3} \Rightarrow 18x - 12 + 3x = 6x - 2 \Rightarrow 15x = 10$

$\Rightarrow x = \frac{2}{3} \Rightarrow S = \left\{\frac{2}{3}\right\}$

2.5.9 Résoudre (sans formule) les équations ci-dessous.

Pas Δ !

a) $x^2 - 9 = 0$

b) $x^2 + 5x = 0$

c) $(x-3)^2 = 0$

d) $(x-3)^2 - 4 = 0$

e) $4(x+5)^2 - 9 = 0$

f) $4(x+5)^2 + 9 = 0$

g) $(x-p)^2 - q = 0$

h) $x^2 + 6x + 9 - 4 = 0$

i) $x^2 + 6x + 5 = 0$

j) $x^2 + 5x + 4 = 0$

k) $2x^2 + 10x + 8 = 0$

l) $ax^2 + bx + c = 0$

a) $x^2 - 9 = 0$

$\Leftrightarrow (x-3)(x+3) = 0 \Rightarrow S = \{-3; 3\}$

b) $x^2 + 5x = 0$

$\Leftrightarrow x(x+5) = 0 \Rightarrow x = 0$ ou $x = -5 \Rightarrow S = \{-5; 0\}$

c) $(x-3)^2 = 0$

$\Leftrightarrow x - 3 = 0 \Rightarrow x = 3 \Rightarrow S = \{3\}$

d) $(x-3)^2 - 4 = 0$

$\Leftrightarrow (x-3-2)(x-3+2) = 0 \Leftrightarrow (x-5)(x-1) = 0$

$\Rightarrow S = \{1; 5\}$

ou $(x-3)^2 - 4 \Leftrightarrow x - 3 = \pm 2 \Rightarrow$

$x - 3 = 2 \Rightarrow x = 5$

$x - 3 = -2 \Rightarrow x = 1$

e) $4(x+5)^2 - 9 = 0$

$\Leftrightarrow 4(x+5)^2 = 9 \Leftrightarrow 2(x+5) = \pm 3 \Rightarrow$

$2x + 10 = -3 \Rightarrow x = -\frac{13}{2}$

$2x + 10 = 3 \Rightarrow x = -\frac{7}{2}$

$\Rightarrow S = \left\{-\frac{13}{2}; -\frac{7}{2}\right\}$

f) $4(x+5)^2 + 9 = 0$

$\Leftrightarrow 4(x+5)^2 = -9 < 0$ impossible car $4(x+5)^2 > 0$

$\Rightarrow \boxed{S = \emptyset}$

g) $(x-p)^2 - q = 0$

$\Leftrightarrow (x-p)^2 = q$

* 1er cas : si $q < 0$ $\Rightarrow S = \emptyset$
* 2e cas : si $q \geq 0$ $\Rightarrow (x-p) = \pm\sqrt{q}$

$\Rightarrow \boxed{S = \{ p - \sqrt{q} \ ; \ p + \sqrt{q} \}}$

h) $x^2 + 6x + 9 - 4 = 0$

$\Leftrightarrow x^2 + 6x + 9 = 4 \Leftrightarrow (x+3)^2 = 4$

$\Leftrightarrow x + 3 = \pm 2$

* $x + 3 = -2 \Rightarrow x = -5$
* $x + 3 = 2 \Rightarrow x = -1$

$\Rightarrow \boxed{S = \{-5 ; -1\}}$

i) $x^2 + 6x + 5 = 0$

$\Leftrightarrow x^2 + 6x + 4 + 5 - 4 = 0$

$\Leftrightarrow x^2 + 6x + 9 - 4 = 0 \Leftrightarrow (x+3)^2 = 4$

$\Rightarrow x + 3 = \pm 2 \Rightarrow \boxed{S = \{-5 ; -1\}}$

j) $x^2 + 5x + 4 = 0$

$\Leftrightarrow x^2 + 5x + 6.25 - 2.25 = 0$

$\Leftrightarrow \quad (x+2.5)^2 = 2.25$

$\Leftrightarrow \quad x + 2.5 = \pm 1.5 \quad \Rightarrow \boxed{S = \{-4 ; -1\}}$

k) $2x^2 + 10x + 8 = 0 \quad | :(2)$

$\Leftrightarrow \quad x^2 + 5x + 4 = 0$

comme point j) $\Rightarrow \boxed{S = \{-4 ; -1\}}$

l) $ax^2 + bx + c = 0$

* $a \neq 0$ donc équation du 1er degré !

$\Rightarrow \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \Rightarrow x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} + \frac{c}{a} - \frac{b^2}{4a^2} = 0$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

* <u>1er cas :</u> $b^2 - 4ac < 0 \Rightarrow S = \emptyset$

* <u>2e cas :</u> $b^2 - 4ac \geq 0 \Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \boxed{S = \left\{ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right\}}$

2.5.10 Résoudre les équations suivantes.

a) $x^2 - 3x + 2 = 0$

b) $x^2 - 5x + 4 = 0$

c) $x^2 - 4x + 5 = 0$

d) $x^2 + 6x + 9 = 0$

e) $-x^2 + 30x - 209 = 0$

f) $2x^2 - 5x - 2 = 0$

g) $-\frac{1}{2}x^2 + x + 6 = 0$

h) $2x^2 = x + 6$

a) $x^2 - 3x + 2 = 0$

$\Leftrightarrow (x-2)(x-1) = 0 \Rightarrow S = \{1; 2\}$

b) $x^2 - 5x + 4 = 0$

$\Leftrightarrow (x-4)(x-1) = 0 \Rightarrow S = \{1; 4\}$

c) $x^2 - 4x + 5 = 0$

$\Delta = (-4)^2 - 4 \cdot 1 \cdot 5 = 16 - 20 = -4 < 0 \Rightarrow S = \emptyset$

d) $x^2 + 6x + 9 = 0$

$\Leftrightarrow (x+3)^2 = 0 \Rightarrow x = -3 \Rightarrow S = \{-3\}$

e) $-x^2 + 30x - 209 = 0$

$\Leftrightarrow (x-11)(x-19) = 0 \Rightarrow S = \{11; 19\}$

g) $-\frac{1}{2}x^2 + x + 6 = 0$

$\Leftrightarrow -x^2 + 2x + 12 = 0 \Leftrightarrow x^2 - 2x - 12 = 0$

$\Delta = 4 - 4(-12) = 52 \Rightarrow \sqrt{\Delta} = \sqrt{4 \cdot 13} = 2\sqrt{13}$

$\Rightarrow x_1 = \frac{2 - 2\sqrt{13}}{2} = \frac{2(1 - \sqrt{13})}{2} = 1 - \sqrt{13}$; $x_2 = 1 + \sqrt{13}$

$\Rightarrow S = \{1 - \sqrt{13}; 1 + \sqrt{13}\}$

2.5.11 Résoudre les équations suivantes après avoir déterminé leur ensemble de définition.

a) $\dfrac{x-1}{2x-1} = \dfrac{3x-5}{4x-2}$

c) $\dfrac{1}{x+1} + \dfrac{1}{x+3} + \dfrac{3}{4} = 0$

b) $\dfrac{x^2+x+1}{2x+2} = x$

d) $\dfrac{x}{x-1} = \dfrac{3x-4}{(x-1)\cdot(x-2)}$

a) $\dfrac{x-1}{2x-1} = \dfrac{3x-5}{4x-2}$

$\Leftrightarrow \dfrac{x-1}{2x-1} = \dfrac{3x-5}{2(2x-1)}$ (*) $\Rightarrow$ condition : $2x-1 \neq 0 \Rightarrow x \neq \frac{1}{2}$

$\Rightarrow ED = \mathbb{R} \setminus \{\frac{1}{2}\}$

(*) $\Leftrightarrow 2(x-1) = 3x-5 \Rightarrow 2x-2 = 3x-5 \Rightarrow x = 3$ ✓

$\Rightarrow \boxed{S = \{3\}}$

b) $\dfrac{x^2+x+1}{2x+2} = x$ (*)

condition : $2x+2 \neq 0 \Rightarrow x \neq -1 \Rightarrow ED = \mathbb{R} \setminus \{-1\}$

(*) $\Leftrightarrow x^2+x+1 = x(2x+2)$

$\Leftrightarrow x^2+x+1 = 2x^2+2x \Rightarrow x^2+x-1 = 0 \Rightarrow \Delta = 5$

$\Rightarrow x_1 = \dfrac{-1-\sqrt{5}}{2}$, $x_2 = \dfrac{-1+\sqrt{5}}{2}$

$\Rightarrow \boxed{S = \left\{\dfrac{-1-\sqrt{5}}{2}\ ;\ \dfrac{-1+\sqrt{5}}{2}\right\}}$

c) $\frac{1}{x+1} + \frac{1}{x+3} + \frac{3}{4} = 0$ (*)

conditions : $x+1 \neq 0$ $x+3 \neq 0$

$\Rightarrow$ ED = $\mathbb{R} \setminus \{-3 ; -1\}$

(*) $\Leftrightarrow$ $1 \cdot 4 \cdot (x+3) + 1 \cdot 4 \cdot (x+1) + 3(x+1)(x+3) = 0$

$\Leftrightarrow$ $4x + 12 + 4x + 4 + 3x^2 + 12x + 9 = 0$

$\Leftrightarrow$ $3x^2 + 20x + 25 = 0$ $\Leftrightarrow$ $(3x+5)(x+5) = 0$

$\Leftrightarrow$ $x_1 = -5$, $x_2 = -\frac{5}{3}$ $\Rightarrow$ $S = \left\{-5 ; -\frac{5}{3}\right\}$

d) $\frac{x}{x-1} = \frac{3x-4}{(x-1)(x-2)}$ (*)

$\Rightarrow$ ED = $\mathbb{R} \setminus \{1 ; 2\}$

(*) $\Leftrightarrow$ $x(x-2) = 3x-4$ $\Rightarrow$ $x^2 - 5x + 4 = 0$

$\Rightarrow$ $(x-4)(x-1) = 0$ $\Rightarrow$ $x_1 = 1$, $x_2 = 4$

! solution à éliminer (ED)

$\Rightarrow$ $S = \{4\}$

e) $\frac{750}{x} + 6 = \frac{720}{x-5}$

f) $\frac{x}{x-6} - \frac{1}{2} = \frac{x}{6} + \frac{x+6}{6-x}$

e) $\frac{750}{x} + 6 = \frac{720}{x-5}$ (*)

$\Rightarrow$ ED = $\mathbb{R}^* \setminus \{5\}$ (*) $\Leftrightarrow$ $750(x-5) + 6x(x-5) = 720x$

$\Rightarrow$ $750x - 3750 + 6x^2 - 30x = 720x$ $\Rightarrow$ $6x^2 - 3750 = 0$

$\Rightarrow 6(x^2 - 625) = 0 \quad \Rightarrow 6(x-25)(x+25) = 0$

$\Rightarrow x_1 = -25 \; , \; x_2 = 25$

$\Rightarrow \boxed{S = \{-25 ; 25\}}$

f) $\dfrac{x}{x-6} - \dfrac{1}{2} = \dfrac{x}{6} + \dfrac{x+6}{6-x}$ (*)

$\Rightarrow ED = \mathbb{R} \setminus \{6\}$

(*) $\Leftrightarrow \dfrac{x}{x-6} - \dfrac{1}{2} = \dfrac{x}{6} + \dfrac{x+6}{-(x-6)} \quad \Rightarrow \dfrac{x}{x-6} - \dfrac{1}{2} = \dfrac{x}{6} - \dfrac{x+6}{x-6}$

$\Rightarrow 6x - 3x + 18 = x^2 - 6x - 6x - 36$

$\Rightarrow x^2 - 15x - 54 = 0 \quad \Rightarrow (x-18)(x+3) = 0 \quad \Rightarrow x_1 = -3 \; , x_2 = 18$

$\Rightarrow \boxed{S = \{-3 ; 18\}}$

2.5.12 Résoudre les équations ci-dessous :

a) $\dfrac{x-4}{x+8}=0$

b) $\dfrac{g^2-5g}{g^2-8g+15}=0$

c) $\dfrac{2x^3-8x^2-10x}{x-5}=5x$

d) $\dfrac{x+1}{x}-2x=\dfrac{x-1}{x}$

e) $\dfrac{z}{z-3}-\dfrac{2}{2-z}=\dfrac{3}{z^2-5z+6}$

f) $\dfrac{4x}{x+3}-\dfrac{x}{x-3}=-\dfrac{12}{x^2-9}$

g) $\dfrac{t}{t-2}-\dfrac{2}{t+2}=\dfrac{8}{t^2-4}$

h) $\dfrac{x+4}{x}-\dfrac{1}{x+4}=\dfrac{4}{x^2+4x}$

i) $\dfrac{1}{x^2-x}+\dfrac{5}{x^2+x}=\dfrac{4}{x^2-1}$

j) $\dfrac{x+3}{3x-1}+\dfrac{1}{4}=\dfrac{2x-9}{4-12x}+1$

a) $\dfrac{x-4}{x+8}=0 \quad \Leftrightarrow$ ED $= \mathbb{R}\setminus\{-8\}$

$\Leftrightarrow x-4=0 \quad \Leftrightarrow x=4 \quad \Leftrightarrow$ **$S=\{4\}$**

b) $\dfrac{g^2-5g}{g^2-8g+15}=0 \quad (*)$

Condition : $g^2-8g+15\neq 0 \Leftrightarrow (g-3)(g-5)\neq 0$

$\Leftrightarrow$ ED $= \mathbb{R}\setminus\{3;5\}$

$(*) \Leftrightarrow g^2-5g=0 \Leftrightarrow g(g-5)=0 \Leftrightarrow g_1=0,\ g_2=5$

g_2 : solution à éliminer

$\Leftrightarrow$ **$S=\{0\}$**

c) $\dfrac{2x^3-8x^2-10x}{x-5}=5x \quad \Leftrightarrow$ ED $= \mathbb{R}\setminus\{5\}$

$\Leftrightarrow 2x^3-8x^2-10x=5x(x-5) \Leftrightarrow 2x^3-13x^2+15x=0$

$\Leftrightarrow x(2x^2-13x+15)=0 \Leftrightarrow x(2x-3)(x-5)=0$

$\Rightarrow x_1 = 0$, $x_2 = \frac{3}{2}$, $x_3 = 5$

x_3 : solution à éliminer

$\Rightarrow \boxed{S = \{0 ; \frac{3}{2}\}}$

d) $\frac{x+1}{x} - 2x = \frac{x-1}{2}$ $\Rightarrow$ $ED = \mathbb{R}^*$

$\Rightarrow x+1 - 2x \cdot x = x-1 \Rightarrow 2x^2 - 2 = 0 \Rightarrow 2(x^2-1) = 0$

$\Rightarrow 2(x-1)(x+1) = 0 \Rightarrow x_1 = -1$, $x_2 = 1$

$\Rightarrow \boxed{S = \{-1 ; 1\}}$

e) $\frac{z}{z-3} - \frac{2}{2-z} = \frac{3}{z^2-5z+6}$ (*)

$z^2 - 5z + 6 = (z-3)(z-2)$ $\Rightarrow ED = \mathbb{R} \setminus \{2 ; 3\}$

(*) $\frac{z}{z-3} + \frac{2}{z-2} = \frac{3}{(z-3)(z-2)} \Rightarrow z \cdot (z-2) + 2(z-3) = 3$

$\Rightarrow z^2 - 2z + 2z - 6 = 3 \Rightarrow z^2 - 9 = 0 \Rightarrow (z-3)(z+3) = 0$

$\Rightarrow z_1 = -3$, $z_2 = 3$

z_2 : solution à éliminer (ED)

$\Rightarrow \boxed{S = \{-3\}}$

f) $\frac{4x}{x+3} - \frac{x}{x-3} = -\frac{12}{x^2-9}$ (*)

$x^2 - 9 = (x-3)(x+3)$

$ED = \mathbb{R} \setminus \{-3 ; 3\}$

(*) $\frac{4x}{x+3} - \frac{x}{x-3} = -\frac{12}{(x-3)(x+3)}$

$\Leftrightarrow \quad 4x(x-3) - x(x+3) = -12$

$\Leftrightarrow \quad 4x^2 - 12x - x^2 - 3x + 12 = 0 \quad \Leftrightarrow \quad 3x^2 - 15x + 12 = 0$

$\Leftrightarrow \quad x^2 - 5x + 4 = 0 \quad \Leftrightarrow \quad (x-4)(x-1) = 0$

$\Leftrightarrow x_1 = 1 \,, \quad x_2 = 4 \quad \Leftrightarrow \quad \boxed{S = \{1 ; 4\}}$

g) $\dfrac{t}{t-2} - \dfrac{2}{t+2} = \dfrac{8}{t^2-4}$ (*)

$t^2 - 4 = (t-2)(t+2)$

$ED = \mathbb{R} \setminus \{-2 ; 2\}$

(*) $\Rightarrow \quad t(t+2) - 2(t-2) = 8 \quad \Leftrightarrow \quad t^2 + 2t - 2t + 4 = 8$

$\Leftrightarrow \quad t^2 - 4 = 0 \quad \Leftrightarrow \quad (t-2)(t+2) = 0$

$\Leftrightarrow \quad t_1 = -2 \,, \quad t_2 = 2 \quad \Leftrightarrow \quad S = \emptyset$

Solutions à éliminer

h) $\dfrac{x+4}{x} - \dfrac{1}{x+4} = \dfrac{4}{x^2+4x}$ (*)

$x^2 + 4x = x(x+4)$

$\Leftrightarrow ED = \mathbb{R}^* \setminus \{-4\}$

(*) $\Leftrightarrow \quad (x+4)(x+4) - x = 4 \quad \Leftrightarrow \quad x^2 + 7x + 12 = 0$

$\Leftrightarrow \quad (x+4)(x+3) = 0 \quad \Leftrightarrow \quad x_1 = -4 \,, \quad x_2 = -3$

solution à éliminer

$\Leftrightarrow \quad \boxed{S = \{-3\}}$

i) $\frac{1}{x^2-x} + \frac{5}{x^2+x} = \frac{4}{x^2-1}$

$\Leftrightarrow \frac{1}{x(x-1)} + \frac{5}{x(x+1)} = \frac{4}{(x-1)(x+1)}$ (*)

$\Leftrightarrow$ ED $= \mathbb{R}^* \setminus \{-1 ; 1\}$

(*) $\Leftrightarrow x + 1 + 5x - 5 = 4x \Leftrightarrow 2x = 4 \Leftrightarrow x = 2 \Leftrightarrow$ **$S = \{2\}$**

j) $\frac{x+3}{3x-1} + \frac{1}{4} = \frac{2x-9}{4-12x} + 1$ (*)

$= -(12x-4) = -4(3x-1)$

$\Leftrightarrow$ ED $= \mathbb{R} \setminus \{\frac{1}{3}\}$

(*) $\Leftrightarrow \frac{x+3}{3x-1} + \frac{1}{4} = -\frac{2x-9}{4(3x-1)} + 1$

$\Leftrightarrow 4(x+3) + 3x - 1 = -(2x-9) + 4(3x-1)$

$\Leftrightarrow 4x + 12 + 3x - 1 = -2x + 9 + 12x - 4$

$\Leftrightarrow 7x + 11 = 10x + 5 \Leftrightarrow 7x - 10x = 5 - 11$

$\Leftrightarrow -3x = -6 \Leftrightarrow x = 2$

$\Leftrightarrow$ **$S = \{2\}$**

2.5.13 Résoudre les équations ci-dessous :

a) $1 = \dfrac{3x}{x^2-9} - \dfrac{x}{2x-6}$

b) $\dfrac{x}{x+3} = \dfrac{x-1}{2x} + \dfrac{1}{4}$

c) $\dfrac{2-x}{x+1} - \dfrac{5}{3} = \dfrac{2x+1}{3-2x}$

d) $\dfrac{x-3}{x^2-3x+2} - \dfrac{x-1}{x^2-5x+6} + \dfrac{x-2}{x^2-4x+3} = 0$

d) $\dfrac{10x-2}{6x-3} + \dfrac{3x+5}{4x^2-1} = \dfrac{x-1}{2x+1}$

e) $\dfrac{x-1}{x^2+x-6} + \dfrac{2x+1}{x^2-5x+6} + \dfrac{x+5}{x^2-9} = 0$

a) $1 = \dfrac{3x}{x^2-9} - \dfrac{x}{2x-6}$

$\Leftrightarrow 1 = \dfrac{3x}{(x-3)(x+3)} - \dfrac{x}{2(x-3)}$ $\Leftrightarrow$ $ED = \mathbb{R} \setminus \{-3; 3\}$

$\Leftrightarrow 2(x^2-9) = 2 \cdot 3x - x(x+3) \Leftrightarrow 2x^2 - 18 = 6x - x^2 - 3x$

$\Leftrightarrow 3x^2 - 3x - 18 = 0 \quad | \div 3$

$\Leftrightarrow x^2 - x - 6 = 0 \Leftrightarrow (x-3)(x+2) = 0$

$\Leftrightarrow x_1 = -2 \; ; \; x_2 = 3$

$x_2 = 3$ Solution à éliminer

$\Leftrightarrow \boxed{S = \{-2\}}$

b) $\dfrac{x}{x+3} = \dfrac{x-1}{2x} + \dfrac{1}{4}$ $\Leftrightarrow$ $ED = \mathbb{R}^* \setminus \{-3\}$

$\Leftrightarrow 4x \cdot x = 2(x-1)(x+3) + x(x+3)$

$\Leftrightarrow 4x^2 = 2(x^2+3x-x-3) + x^2+3x$

$\Leftrightarrow 4x^2 = 2(x^2+2x-3) + x^2+3x \Leftrightarrow 4x^2 = 2x^2+4x-6+x^2+3x$

$\Leftrightarrow 4x^2 - 3x^2 - 7x + 6 = 0$

$\Leftrightarrow x^2 - 7x + 6 = 0 \Leftrightarrow (x-6)(x-1) = 0$

$\Leftrightarrow x_1 = 1 \; , \; x_2 = 6 \Leftrightarrow \boxed{S = \{1; 6\}}$

c) $\frac{2-x}{x+1} - \frac{5}{3} = \frac{2x+1}{3-2x}$ (*)

$ED = \mathbb{R} \setminus \{-1; \frac{3}{2}\}$

$(*) \Leftrightarrow 3 \cdot (3-2x)(2-x) - 5(x+1)(3-2x) = 3(x+1)(2x+1)$

$\Leftrightarrow 3(6 - 3x - 4x + 2x^2) - 5(3x - 2x^2 + 3 - 2x) = 3(2x^2 + x + 2x + 1)$

$\Leftrightarrow 3(2x^2 - 7x + 6) - 5(-2x^2 + x + 3) = 3(2x^2 + 3x + 1)$

$\Leftrightarrow 6x^2 - 21x + 18 + 10x^2 - 5x - 15 = 6x^2 + 9x + 3$

$\Leftrightarrow 16x^2 - 26x + 3 - 6x^2 - 9x - 3 = 0$

$\Leftrightarrow 10x^2 - 35x = 0 \Leftrightarrow 5x(2x - 7) = 0$

$\Leftrightarrow x_1 = 0 \; , \; x_2 = \frac{7}{2}$

$\Leftrightarrow \boxed{S = \{0; \frac{7}{2}\}}$

2.5.14 Résoudre les équations suivantes.

a) $\sqrt{7-x} = x-5$

b) $x = 4 + \sqrt{4x-19}$

c) $\sqrt{x+1} - x = x+2$

d) $x - \sqrt{-7x-24} = -2$

~~k)~~ $\sqrt{x+2} + \sqrt{x-2} = \sqrt{2x+3} + \sqrt{2x-3}$

~~l)~~ $\sqrt{3x-5} - \sqrt{x-1} = \sqrt{4x-5} - \sqrt{2x-1}$

e) $\sqrt{7-2x} - \sqrt{5+x} = \sqrt{4+3x}$

f) $\sqrt{11+8x} + 1 = \sqrt{9+4x}$

g) $x + \sqrt{x} = 20$

h) $\dfrac{\sqrt{3}+\sqrt{x}}{\sqrt{3}-\sqrt{x}} = 3$

a) $\sqrt{7-x} = x-5$

Conditions d'existence : $7-x \geq 0 \Rightarrow x \leq 7$

$x-5 \geq 0 \Rightarrow x \geq 5$

$\Rightarrow$ ED $= [5 ; 7]$

$\sqrt{7-x} = x-5 \quad | \,()^2$

$\Rightarrow 7-x = (x-5)^2 \Rightarrow 7-x = x^2 - 10x + 25$

$\Rightarrow x^2 - 9x + 18 = 0 \Rightarrow (x-3)(x-6) = 0$

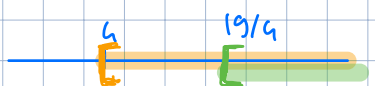
$\Rightarrow x_1 = 3$, $x_2 = 6$ $\Rightarrow S = \{6\}$

Solution à éliminer ($\notin$ ED)

b) $x = 4 + \sqrt{4x-19} \Leftrightarrow x-4 = \sqrt{4x-19}$

Conditions d'existence : $x-4 \geq 0 \Rightarrow x \geq 4$

$4x-19 \geq 0 \Rightarrow x \geq \frac{19}{4}$



$\Rightarrow$ ED $= \left[\frac{19}{4} ; +\infty\right[$

$\Leftrightarrow \quad x-4 = \sqrt{4x-19} \quad | \,()^2$

$\Leftrightarrow \quad (x-4)^2 = 4x-19$

$\Leftrightarrow \quad x^2 - 8x + 16 - 4x + 19 = 0 \quad \Leftrightarrow \quad x^2 - 12x + 35 = 0$

$\Rightarrow (x-5)(x-7) = 0 \quad \Leftrightarrow \quad x_1 = 5 \;,\; x_2 = 7$

$\Leftrightarrow \boxed{S = \{5\,;\,7\}}$

c) $\sqrt{x+1} - x = x+2$

$\Leftrightarrow \quad \sqrt{x+1} = 2x+2 \qquad (*)$

conditions : $\quad x+1 \geqslant 0 \quad \Leftrightarrow \quad x \geqslant -1$

$\Leftrightarrow ED = [-1\,;+\infty[$

$2x+2 \geqslant 0 \quad \Leftrightarrow \quad x \geqslant -1$

$(*) \quad \sqrt{x+1} = 2x+2 \quad | \,()^2$

$x+1 = (2x+2)^2 \quad \Leftrightarrow \quad x+1 = 4x^2 + 8x + 4$

$\Leftrightarrow \quad 4x^2 + 7x + 3 = 0 \quad \Leftrightarrow \quad (4x+3)(x+1) = 0$

$\Leftrightarrow \quad x_1 = -1 \;,\; x_2 = -\frac{3}{4} \quad \Leftrightarrow \boxed{S = \left\{-1\,;-\frac{3}{4}\right\}}$

d) $x - \sqrt{-7x-24} = -2$

$\Leftrightarrow \quad x+2 = \sqrt{-7x-24}$

conditions : $\quad x+2 \geqslant 0 \quad \Leftrightarrow \quad x \geqslant -2 \quad \Leftrightarrow \quad x \geqslant -2$

$-7x-24 \geqslant 0 \quad \Leftrightarrow \quad -7x \geqslant 24 \quad \Leftrightarrow \quad x \leqslant -\frac{24}{7}$

$\Leftrightarrow ED = \emptyset \quad \Leftrightarrow \boxed{S = \emptyset}$

e) $\sqrt{7-2x} - \sqrt{5+x} = \sqrt{4+3x}$

$\Rightarrow \sqrt{7-2x} = \sqrt{4+3x} + \sqrt{5+x}$ (*)

conditions : $7-2x \geq 0 \Rightarrow 7 \geq 2x \Rightarrow x \leq \frac{7}{2}$

$4+3x \geq 0 \Rightarrow 3x \geq -4 \Rightarrow x \geq -\frac{4}{3}$

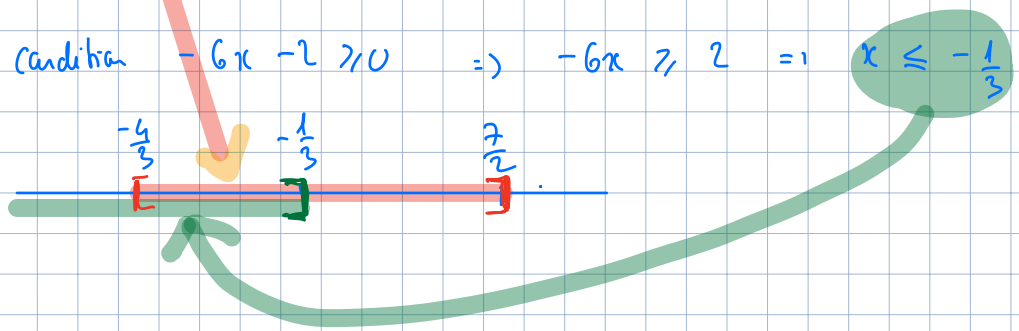
$5+x \geq 0 \Rightarrow x \geq -5 \Rightarrow x \geq -5$

$\rightarrow$ ED $= \left[-\frac{4}{3} ; \frac{7}{2}\right]$

(*) $\Rightarrow 7-2x = 4+3x + 2\sqrt{4+3x}\sqrt{5+x} + 5+x$

$\Rightarrow -6x-2 = 2\sqrt{4+3x}\sqrt{5+x}$ (**)

condition $-6x-2 \geq 0 \Rightarrow -6x \geq 2 \Rightarrow x \leq -\frac{1}{3}$



$\Rightarrow$ ED$_{final}$ $= \left[-\frac{4}{3} ; -\frac{1}{3}\right]$

(**) $-6x-2 = 2\sqrt{4+3x}\sqrt{5+x} \quad | \div 2$

$\Rightarrow -3x-1 = \sqrt{4+3x}\sqrt{5+x} \quad | ()^2$

$(-3x-1)^2 = (4+3x)(5+x) \Rightarrow 9x^2+6x+1 = 3x^2+19x+20$

$\Rightarrow 6x^2 - 13x - 19 = 0 \Rightarrow (6x-19)(x+1) = 0$

$\Rightarrow x_1 = -1$, $x_2 = \frac{19}{6}$ $\Rightarrow S = \{-1\}$

solution à éliminer ($\notin$ ED$_{final}$)

$\S 1$ $\sqrt{11+8x} + 1 = \sqrt{9+4x}$ (*)

conditions : $11 + 8x \geq 0 \iff x \geq -\frac{11}{8}$

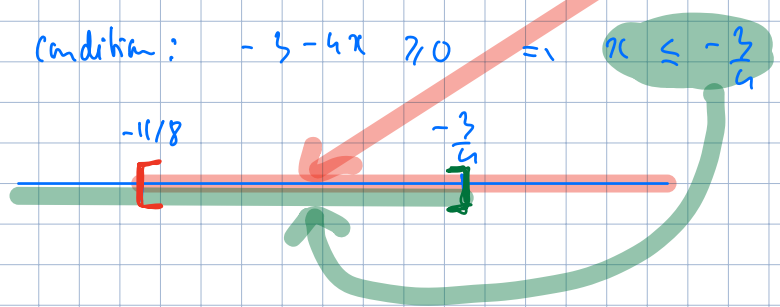
$9 + 4x \geq 0 \iff x \geq -\frac{9}{4}$

$\Rightarrow$ ED $= \left[-\frac{11}{8} ; +\infty\right[$

$\sqrt{11+8x} + 1 = \sqrt{9+4x} \quad | \ ()^2$

$11 + 8x + 2\sqrt{11+8x} + 1 = 9 + 4x \iff 2\sqrt{11+8x} = -3 - 4x$ (**)

$\Rightarrow$ condition : $-3 - 4x \geq 0 \iff x \leq -\frac{3}{4}$



$\Rightarrow$ ED final $= \left[-\frac{11}{8} ; -\frac{3}{4}\right]$

(**) $2\sqrt{11+8x} = -3 - 4x \quad | \ ()^2$

$4(11+8x) = (-3-4x)^2 \iff 4(11+8x) = 9 + 24x + 16x^2$

$\Rightarrow 16x^2 - 8x - 55 = 0$

$\Rightarrow (4x - 7)(4x + 5) = 0$

$\Rightarrow x_1 = -\frac{5}{4}$ | $x_2 = \frac{7}{4}$

x_2 : solution à éliminer ($\notin$ ED final)

$\Rightarrow \boxed{S = \left\{-\frac{5}{4}\right\}}$

2.5.15 Résoudre les équations suivantes.

a) $3\sqrt{2x-3}+2\sqrt{7-x}=11$ ~~b)~~ $\sqrt{2\sqrt{x+1}}=\sqrt{3x-5}$

a) $3\sqrt{2x-3}+2\sqrt{7-x}=11$

conditions : $\begin{cases} 2x-3 \geq 0 \\ 7-x \geq 0 \end{cases} \Leftrightarrow \begin{cases} x \geq \frac{3}{2} \\ x \leq 7 \end{cases} \Leftrightarrow ED = \left[\frac{3}{2} ; 7\right]$

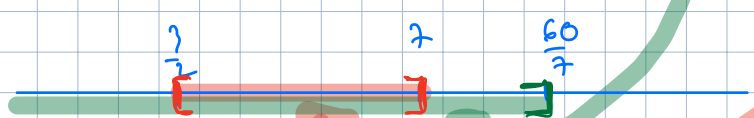
$3\sqrt{2x-3}+2\sqrt{7-x}=11 \quad |(\,)^2$

$\Leftrightarrow 9(2x-3)+12\sqrt{2x-3}\sqrt{7-x}+4(7-x)=121$

$\Leftrightarrow 12\sqrt{2x-3}\sqrt{7-x}=-14x+120 \quad |\div(2)$

$\Leftrightarrow 6\sqrt{2x-3}\sqrt{7-x}=-7x+60 \quad (**)$

conditions : $-7x+60 \geq 0 \Leftrightarrow x \leq \frac{60}{7}$



$\Leftrightarrow ED_{final} = \left[\frac{3}{2} ; 7\right] = ED$

$6\sqrt{2x-3}\sqrt{7-x}=-7x+60 \quad |(\,)^2$

$36(2x-3)(7-x)=(-7x+60)^2 \Leftrightarrow -72x^2+612x-756=49x^2-840x+3600$

$\Leftrightarrow 121x^2-1452x+4536=0 \Leftrightarrow x^2-12x+36=0$

$\Leftrightarrow (x-6)^2=0 \Leftrightarrow x=6 \Leftrightarrow S=\{6\}$

2.5.18 Résoudre les équations suivantes.

a) $|x+4| = 11$

b) $3\,|x-2|+3 = 7$

c) $4 - |x+2| = 3\,(|x-1|-1)$

d) $|2\,x+3| - |2-x| = -3$

a) $|x+4| = 11$

En utilisant la définition de la valeur absolue, on a :

$x+4 = \pm 11 \Leftrightarrow \begin{cases} x+4 = 11 \\ x+4 = -11 \end{cases} \Leftrightarrow \begin{cases} x = 7 \\ x = 15 \end{cases}$

$\Leftrightarrow \boxed{S = \{7; 15\}}$

b) $3|x-2|+3 = 7$

$\Leftrightarrow 3|x-2| = 4 \Leftrightarrow 3(x-2) = \pm 4$

$\Leftrightarrow \begin{cases} 3x-6 = 4 \\ 3x-6 = -4 \end{cases} \Leftrightarrow \begin{cases} 3x = 10 \\ 3x = 2 \end{cases} \Leftrightarrow \begin{cases} x = \frac{10}{3} \\ x = \frac{2}{3} \end{cases}$

$\Leftrightarrow \boxed{S = \left\{\frac{2}{3}; \frac{10}{3}\right\}}$

c) $4 - |x+2| = 3(|x-1|-1)$

$\Leftrightarrow 4 = 3|x-1| - 3 + |x+2|$

$\Leftrightarrow 7 = |x+2| + 3|x-1|$

* téro de $x+2$: $x+2 = 0 \Leftrightarrow x = -2$

* téro de $x-1$: $x-1 = 0 \Leftrightarrow x = 1$

* Définition :

$$|x+2| = \begin{cases} x+2 & \text{si } x+2 \geq 0 \Leftrightarrow x \geq -2 \\ -(x+2) & \text{si } x+2 < 0 \Leftrightarrow x < -2 \end{cases}$$

$$|x-1| = \begin{cases} x-1 & \text{si } x-1 \geq 0 \Leftrightarrow x \geq 1 \\ -(x-1) & \text{si } x-1 < 0 \Leftrightarrow x < 1 \end{cases}$$

* Tableau :

x	$-\infty$		-2		1		$+\infty$
$\lvert x+2 \rvert$	$-(x+2)$		0	$x+2$		$x+2$	
$\lvert x-1 \rvert$	$-(x-1)$		$-(x-1)$		0	$x-1$	
Equations	$7 = -(x+2)+3[-(x-1)]$ cas 3			$7 = x+2+3[-(x-1)]$ cas 2		$7 = x+2+3(x-1)$ cas 1	

* cas 1 : $x \geq 1$

$\Leftrightarrow 7 = x+2+3(x-1) \Leftrightarrow 7 = x+2+3x-3$

$\Leftrightarrow 4x = 8 \Leftrightarrow x = 2$ solution OK

* cas 2 : $-2 \leq x < 1$

$\Leftrightarrow 7 = x+2+3[-(x-1)] \Leftrightarrow 7 = x+2-3(x-1)$

$\Leftrightarrow 7 = x+2-3x+3 \Leftrightarrow -2x = 2 \Leftrightarrow x = -1$ solution OK

* cas 3 : $x < -2$

$\Leftrightarrow 7 = -(x+2)+3[-(x-1)] \Leftrightarrow 7 = -x-2-3(x-1)$

$\Leftrightarrow 7 = -x-2-3x+3 \Leftrightarrow -4x = 6 \Leftrightarrow x = -\frac{3}{2}$

Solution à éliminer

$\Leftrightarrow \boxed{S = \{-1 ; 2\}}$

d) $|2x+3| - |2-x| = -3$

* Zéro de $2x+3$: $2x+3=0 \Leftrightarrow x = -\frac{3}{2}$

* Zéro de $2-x$: $2-x=0 \Leftrightarrow x = 2$

* Définition :

$$|2x+3| = \begin{cases} 2x+3 & \text{si } 2x+3 \geq 0 \Leftrightarrow x \geq -\frac{3}{2} \\ -(2x+3) & \text{si } 2x+3 < 0 \Leftrightarrow x < -\frac{3}{2} \end{cases}$$

$$|2-x| = \begin{cases} 2-x & \text{si } 2-x \geq 0 \Leftrightarrow x \leq 2 \\ -(2-x) & \text{si } 2-x < 0 \Leftrightarrow x > 2 \end{cases}$$

* Tableau :

x	$-\infty$		$-\frac{3}{2}$		2		$+\infty$
$\lvert 2x+3 \rvert$	$-(2x+3)$		0	$2x+3$		$2x+3$	
$\lvert 2-x \rvert$	$2-x$		$2-x$		0	$-(2-x)$	
Equations	$-(2x+3)-(2-x)=-3$		$2x+3-(2-x)=-3$			$2x+3-(-(2-x))=-3$	
	cas 3		cas 2			cas 1	

* <u>cas 1</u> : $x > 2$

$2x+3-(-(2-x)) = -3$

$\Leftrightarrow 2x+3+(2-x) = -3 \Leftrightarrow 2x+3+2-x = -3$

$\Leftrightarrow x = -8$ Solution à éliminer

* <u>cas 2</u> : $-\frac{3}{2} \leq x \leq 2$

$\Leftrightarrow 2x+3-(2-x) = -3 \Leftrightarrow 2x+3-2+x = -3$

$\Leftrightarrow 3x = -4 \Leftrightarrow x = -\frac{4}{3}$ solution ok

* Cas 3 : $x < -\frac{3}{2}$

$\Leftrightarrow -(2x+3) - (2-x) = -3$

$\Leftrightarrow -2x - 3 - 2 + x = -3$

$\Leftrightarrow -x = 2 \Leftrightarrow x = -2$ solution OK

Donc $S = \left\{ -2 ; -\frac{4}{3} \right\}$

2.5.19 Résoudre les systèmes d'équations :

a) $\begin{cases} x+y &= 1 \\ x-y &= 0 \end{cases}$

b) $\begin{cases} 5x-2y &= 5 \\ 3x-y &= 10 \end{cases}$

c) $\begin{cases} 6x+4 &= -6y \\ 1-x &= 6y \end{cases}$

d) $\begin{cases} 2x-4y &= 2 \\ x-2y &= 1 \end{cases}$

e) $\begin{cases} 2x+4y &= 5 \\ x+2y &= 2 \end{cases}$

f) $\begin{cases} 2x+3y &= 4 \\ 5x+6y &= 10 \end{cases}$

a) $\begin{cases} x+y = 1 \\ x-y = 0 \end{cases}$ +

$2x = 1 \Leftrightarrow x = \frac{1}{2} \Leftrightarrow y = \frac{1}{2} \Leftrightarrow S = \left\{ \left(\frac{1}{2} ; \frac{1}{2} \right) \right\}$

b) $\begin{cases} 5x-2y = 5 \\ 3x-y = 10 \end{cases} \Leftrightarrow \left| \cdot (-2) \right.$

$\Leftrightarrow \begin{cases} 5x-2y = 5 \\ -6x+2y = -20 \end{cases}$ +

$-x = -15 \Leftrightarrow x = 15$

$\Leftrightarrow y = 45 - 10 = 35 \Leftrightarrow S = \{ (15 ; 35) \}$

c) $\begin{cases} 6x + 4 = -6y \\ 1 - x = 6y \end{cases}$ $\Leftrightarrow$ $+\begin{cases} 6x + 6y = -4 \\ -x - 6y = -1 \end{cases}$

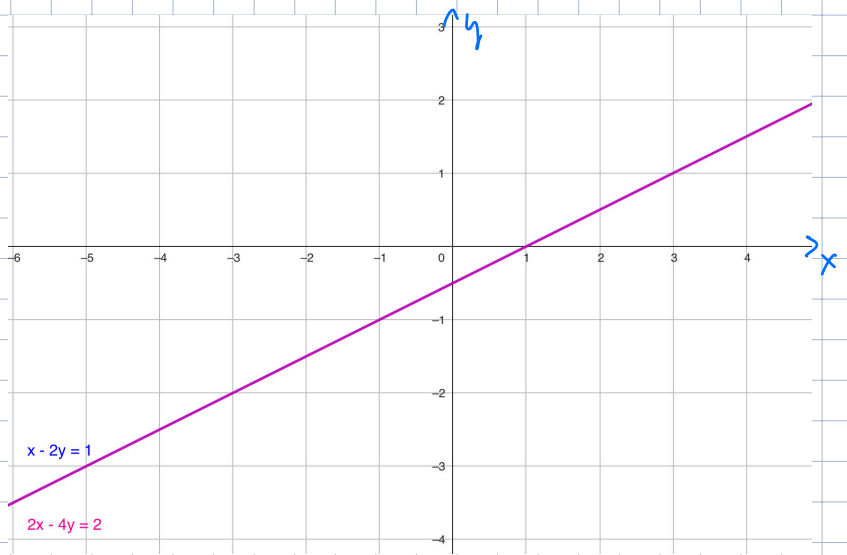
$5x = -5 \Leftrightarrow x = -1$

$\Leftrightarrow y = \frac{1}{3}$ $\Leftrightarrow$ $\boxed{S = \left\{ \left(-1 ; \frac{1}{3} \right) \right\}}$

d) $\begin{cases} 2x - 4y = 2 \\ x - 2y = 1 \end{cases} \Big| \div (-2)$ $\Leftrightarrow$ $+\begin{cases} -x + 2y = -1 \\ x - 2y = 1 \end{cases}$

$0 = 0$

$\Leftrightarrow$ 2 droites confondues



$\Leftrightarrow$ $\boxed{S = \{ (x;y) \in \mathbb{R}^2 \mid x - 2y = 1 \}}$

e) $\begin{cases} 2x + 4y = 5 \\ x + 2y = 2 \end{cases} \Big| \cdot (-2)$ $\Leftrightarrow$ $+\begin{cases} 2x + 4y = 5 \\ -2x - 4y = -4 \end{cases}$

$0 = 1 \Leftrightarrow \boxed{S = \emptyset}$

2.5.20 Pour quelle(s) valeur(s) de m le système suivant admet-il exactement une solution ?

$$\begin{cases} 2x + 5y &= 32 \\ x + y &= 10 \\ 7x - 3y &= m \end{cases}$$

Résoudre d'abord système : $\begin{cases} 2x + 5y = 32 \\ x + y = 10 \end{cases} \Big| \cdot(-2) \Rightarrow \begin{cases} 2x + 5y = 32 \\ -2x - 2y = -20 \end{cases}$ (+)

$3y = 12$

$\Rightarrow y = 4 \quad \Rightarrow x = 6$

Ensuite, remplacer x et y dans $7x - 3y = m$ (car exactement 1 solution)

$\Rightarrow 7 \cdot 6 - 3 \cdot 4 = m \Rightarrow \boxed{m = 30}$

2.5.21 Résoudre les systèmes linéaires ci-dessous :

a) $\begin{cases} 12x - 5y = 29 \\ 4x - 3y = 11 \end{cases}$

b) $\begin{cases} x + y = 19 \\ 2x - 3y = 11 \end{cases}$

c) $\begin{cases} 12x + 11y = 6 \\ 3y - 2x = 24 \end{cases}$

d) $\begin{cases} 72x + 14y = 330 \\ 63x + 7y = 273 \end{cases}$

e) $\begin{cases} 2x + 3y = 4 \\ 3y + 10x = 40 \end{cases}$

f) $\begin{cases} 21x + 8y + 66 = 0 \\ 28x - 23y - 13 = 0 \end{cases}$

g) $\begin{cases} 2x + 3y + 2z = 41 \\ 8x + 5y = 31 \\ 7y = 21 \end{cases}$

h) $\begin{cases} 2x - 3y + 2z = 41 \\ 5x + 3y = 10 - z \\ 9x = 27 \end{cases}$

i) $\begin{cases} 7x - 4y - 5z = 56 \\ 3y - 2z = 13 \\ 5x - 3y = 22 \end{cases}$

j) $\begin{cases} x + y + z = 25 \\ x - y + z = 5 \\ x + 2z = 2y - 10 \end{cases}$

k) $\begin{cases} x - y - z = 6 \\ x - 2y - 3z = 10 \\ 5x + 6y + z = 2 \end{cases}$

c) $\begin{cases} 12x + 11y = 6 \\ 3y - 2x = 24 \end{cases} \Leftrightarrow \begin{cases} 12x + 11y = 6 \\ -2x + 3y = 24 \end{cases} \mid \cdot(6)$

$\Leftrightarrow + \begin{cases} 12x + 11y = 6 \\ -12x + 18y = 144 \end{cases}$

$29y = 150 \Leftrightarrow y = \frac{150}{29} \Leftrightarrow x = \frac{3}{2} \cdot \frac{150}{29} - 12 = -\frac{123}{50}$

$\Leftrightarrow \boxed{S = \left\{ \left(-\frac{123}{50} ; \frac{150}{29} \right) \right\}}$

e) $\begin{cases} 2x + 3y = 4 \\ 3y + 10x = 40 \end{cases} \Leftrightarrow - \begin{cases} 2x + 3y = 4 \\ 10x + 3y = 40 \end{cases}$

$-8x = -36 \Leftrightarrow x = \frac{-36}{-8} = \frac{9}{2}$

$\Rightarrow y = \frac{1}{3}(4 - 9) = -\frac{5}{3} \Leftrightarrow \boxed{S = \left\{ \left(\frac{9}{2} ; -\frac{5}{3} \right) \right\}}$

g) $\begin{cases} 2x + 3y + 2z = 41 \\ 8x + 5y = 31 \\ 7y = 21 \end{cases} \Rightarrow \begin{cases} 2x + 3y + 2z = 41 \\ 8x + 5y = 31 \\ y = 3 \end{cases}$

$\Rightarrow 8x + 5y = 31$

$8x + 5 \cdot 3 = 31 \Rightarrow 8x = 31 - 15 = 16 \Rightarrow x = 2$

$\Rightarrow 2z = 41 - 2x - 3y = 41 - 2 \cdot 2 - 3 \cdot 3 = 41 - 4 - 9 = 28$

$\Rightarrow z = 14$

$\Rightarrow \boxed{S = \{(2; 3; 14)\}}$

h) $\begin{cases} 2x - 3y + 2z = 41 \\ 5x + 3y = 10 - z \\ 9x = 27 \end{cases} \Rightarrow \begin{cases} 2x - 3y + 2z = 41 \\ 5x + 3y + z = 10 \\ x = 3 \end{cases} \Rightarrow \begin{cases} 6 - 3y + 2z = 41 \\ 15 + 3y + z = 10 \\ x = 3 \end{cases}$

$\Rightarrow + \begin{cases} -3y + 2z = 35 \\ 3y + z = -5 \end{cases}$

$3z = 30 \Rightarrow z = 10 \Rightarrow y = \frac{-5 - 10}{3} = -5$

$\Rightarrow \boxed{S = \{(3; -5; 10)\}}$

i) $\begin{cases} 7x - 4y - 5z = 56 \\ 3y - 2z = 13 \\ 5x - 7y = 22 \end{cases} \quad \begin{matrix} \cdot (2) \\ \cdot (-5) \end{matrix}$

$$\Rightarrow + \begin{cases} 14x - 8y - 10z = 112 \\ -15y + 10z = -65 \end{cases}$$

$$14x - 23y = 47$$

$\Rightarrow$ Nouveau système : $\begin{cases} 14x - 23y = 47 \\ 5x - 3y = 22 \end{cases} \begin{matrix} \\ \cdot(-14) \end{matrix}$

$$\Rightarrow \begin{cases} 70x - 115y = 235 \\ -70x + 42y = -308 \end{cases}$$

$$-73y = -73 \Rightarrow y = 1$$

$$\Rightarrow x = \frac{22+3}{5} = 5 \quad \Rightarrow z = \frac{-13+3}{2} = -5$$

$$\Rightarrow \boxed{S = \{(5; 1; -5)\}}$$

j) $\begin{cases} x + y + z = 25 \\ x - y + z = 5 \\ x + 2y = 2y - 10 \end{cases} \Rightarrow \begin{cases} x + y + z = 25 \\ x - y + z = 5 \\ x - 2y + 2z = -10 \end{cases} \begin{matrix} \cdot(1) \\ \cdot(-1) \\ \\ \end{matrix}$

$$\Rightarrow + \begin{cases} x + y + z = 25 \\ -x + y - z = -5 \end{cases}$$

$$2y = 10 \Rightarrow y = 10$$

$$\Rightarrow \begin{cases} x + 10 + z = 25 \\ x - 20 + 2z = -10 \end{cases} \Rightarrow \begin{cases} x + z = 15 \\ x + 2z = 10 \end{cases} \cdot(-1)$$

$$\Rightarrow \begin{cases} x + z = 15 \\ -x - 2z = -10 \end{cases}$$

$$-z = 5 \Rightarrow z = -5 \Rightarrow x = 20$$

$$\Rightarrow \boxed{S = \{(20; 10; -5)\}}$$

k) $\begin{cases} x - y - z = 6 \\ x - 2y - 3z = 10 \\ 5x + 6y + z = 2 \end{cases}$ $\begin{array}{l} \cdot(-3) \\ \cdot(1) \\ \cdot(1) \end{array}$

$\Rightarrow$ $\begin{cases} -3x + 3y + 3z = -18 \\ x - 2y - 3z = 10 \end{cases}$ $+$

$-2x + y = -8$

$+ \begin{cases} x - y - z = 6 \\ 5x + 6y + z = 2 \end{cases}$

$6x + 5y = 8$

$\Rightarrow$ nouveau système : $\begin{cases} -2x + y = -8 \\ 6x + 5y = 8 \end{cases}$ $\begin{array}{l} \cdot(3) \\ \cdot(1) \end{array}$ $\Rightarrow + \begin{cases} -6x + 3y = -24 \\ 6x + 5y = 8 \end{cases}$

$8y = -16$

$\Rightarrow y = -2 \quad \Rightarrow x = 3 \quad \Rightarrow z = -1$

$\Rightarrow S = \{(3; -2; -1)\}$

l) $\begin{cases} 3z - 2y - x = 17 \\ 2y + 3z - 2x = 36 \\ 5x + 2y - z = 10 \end{cases}$

m) $\begin{cases} 3x - y + z = 29 \\ x + 3y + 30z = 6 \\ x - y + z = 17 \end{cases}$

n) $\begin{cases} 2x + 3y + 4z = 47 \\ 3x + 5y - 4z = 2 \\ 4x + 7y - 2z = 31 \end{cases}$

o) $\begin{cases} 2x + y - z = 1 \\ x + 2y + z = 8 \\ 3x - y + 2z = 7 \end{cases}$

p) $\begin{cases} x + 2y - 3z = 0 \\ 5x - 3y + z = 0 \end{cases}$

q) $\begin{cases} 2x - y + 3z = 4 \\ 3x + 4y - z = -5 \\ x + 5y - 4z = -9 \end{cases}$

r) $\begin{cases} x - y + z = 0 \\ -x + y + z = 10 \\ x + y - z = 2 \end{cases}$

s) $\begin{cases} x + 2y + 3z = 9 \\ x - y + 4z = 15 \\ -x + 7y - 6z = -27 \end{cases}$

t) $\begin{cases} x + y + z = 9 \\ x + 2y + 3z = 14 \\ 3x + 2y + z = 22 \end{cases}$

u) $\begin{cases} 3x + 4y - z = -3 \\ 2x + y - z = -1 \\ x + 2y + z = 1 \\ x + y - 3z = -6 \end{cases}$

v) $\begin{cases} 2x + y + 3z = 3 \\ 3x - y + 4z = 2 \\ 4x + y - z = 5 \\ x + y + z = 4 \end{cases}$

w) $\begin{cases} x + y + 2z = 4 \\ 3x - y + z = 3 \end{cases}$

~~x)~~ $\begin{cases} x - 3y + z - t = 0 \\ 2x + y - z + 2t = 0 \end{cases}$

~~x)~~ $\begin{cases} x + 5y + 4z - 13t = 3 \\ 3x - y + 2z + 5t = 2 \\ 2x + 2y + 3z - 4t = 1 \end{cases}$

z) $\begin{cases} x + y + z + t + u = 1 \\ x + 2y + z + t + u = 0 \\ x + y + 3z + t + u = 3 \\ x + y + z + 4t + u = -2 \\ x + y + z + t + 5u = 5 \end{cases}$

m) $\begin{cases} 3x - y + z = 29 \\ x + 3y + 3z = 6 \\ x - y + z = 17 \end{cases} \quad \begin{matrix} \cdot(1) \\ \\ \cdot(-1) \end{matrix}$

$\Leftrightarrow + \begin{cases} 3x - y + z = 29 \\ -x + y - z = -17 \end{cases}$

$2x = 12$

$\Leftrightarrow x = 6$

$\Leftrightarrow \begin{cases} 3 \cdot 6 - y + z = 29 \\ 6 + 3y + 3z = 6 \end{cases}$

$\Leftrightarrow + \begin{cases} -y + z = 11 \\ y + 10z = 0 \end{cases}$

$11z = 11 \Leftrightarrow z = 1$

$\Leftrightarrow y = 1 - 11 = -10$

$\Rightarrow \boxed{S = \{(6 ; -10 ; 1)\}}$

o) $\begin{cases} 2x + y - z = 1 \\ x + 2y + z = 8 \\ 3x - y + 2z = 7 \end{cases} \quad \begin{matrix} \cdot(1) \\ \cdot(1) \\ \\ \cdot(1) \end{matrix} \quad \begin{matrix} \cdot(2) \\ \\ \\ \end{matrix}$

$\Leftrightarrow \begin{cases} 2x + y - z = 1 \\ x + 2y + z = 8 \end{cases}$

$3x + 3y = 9$

et $\begin{cases} 4x + 2y - 2z = 2 \\ 3x - y + 2z = 7 \end{cases}$

$7x + y = 9$

nouveau système : $\begin{cases} 3x + 3y = 9 \\ 7x + y = 9 \end{cases} \quad \begin{matrix} \cdot(1) \\ \cdot(-3) \end{matrix} \Rightarrow \begin{cases} 3x + 3y = 9 \\ -21x - 3y = -27 \end{cases}$

$-18x = -18$

$\Leftrightarrow x = +1 \quad \Leftrightarrow y = 3 - 1 = 2 \quad \Leftrightarrow z = 2 + 2 - 1 = 3$

$\Leftrightarrow \boxed{S = \{(1 ; 2 ; 3)\}}$

p) $\begin{cases} x + 2y - 3z = 0 \\ 5x - 3y + z = 0 \end{cases} \quad \begin{matrix} \cdot(1) \\ \cdot(3) \end{matrix} \Rightarrow \begin{cases} x + 2y - 3z = 0 \\ 15x - 9y + 3z = 0 \end{cases}$

$16x - 7y = 0$

$\Rightarrow x = \frac{7}{16} y$

Remplacer x dans $x + 2y - 3z = 0 \Rightarrow \frac{7}{16} y + 2y - 3z = 0$

$\Rightarrow 3z = \frac{7}{16} y + 2y = \frac{7y + 32y}{16} = \frac{39y}{16}$

$\Rightarrow z = \frac{39y}{3 \cdot 16} = \frac{13y}{16}$

Si $\frac{y}{16} = t \Rightarrow y = 16t$ et $x = 7t$, $z = 13t$

Donc $S = \{ (7t ; 16t ; 13t) \in \mathbb{R}^3 \mid t \in \mathbb{R} \}$

q) $\begin{cases} 2x - y + 3z = 4 \\ 3x + 4y - z = -5 \\ x + 5y - 4z = -9 \end{cases} \quad \begin{matrix} \cdot(1) \\ \cdot(3) \\ \cdot(-4) \\ \cdot(1) \end{matrix}$

$\Rightarrow \begin{cases} 2x - y + 3z = 4 \\ 9x + 12y - 3z = -15 \end{cases} \qquad \begin{cases} -12x - 16y + 4z = 20 \\ x + 5y - 4z = -9 \end{cases}$

$11x + 11y = -11 \qquad -11x - 11y = 11$

$\Rightarrow x + y = -1 \Rightarrow y = -x - 1 \Rightarrow z = 3x + 4y + 5$

$\Rightarrow z = 3x + 4(-x - 1) + 5 = 3x - 4x - 4 + 5 = -x + 1$

Si $x = t \Rightarrow y = -t - 1$, $z = -t + 1$

$\Rightarrow S = \{ (t ; -t - 1 ; -t + 1) \in \mathbb{R}^3 \mid t \in \mathbb{R} \}$

s) $\begin{cases} x + 2y + 3z = 9 \\ x - y + 4z = 15 \\ -x + 7y - 6z = -27 \end{cases}$ $\begin{matrix} \cdot(1) \\ \cdot(-1) \end{matrix}$ $\begin{matrix} \cdot(1) \\ \\ \cdot(1) \end{matrix}$

$+ \begin{cases} x + 2y + 3z = 9 \\ -x + y - 4z = -15 \end{cases}$

$3y - z = -6$

$+ \begin{cases} x + 2y + 3z = 9 \\ -x + 7y - 6z = -27 \end{cases}$

$9y - 3z = -18$

$\Rightarrow 3y - z = -6 \Rightarrow z = 3y + 6$

$\Rightarrow x = 9 - 2y - 3z = 9 - 2y - 3(3y + 6)$

$\Rightarrow x = 9 - 2y - 9y - 18 \Rightarrow x = -11y - 9$

Fie $y = t \Rightarrow x = -11t - 9$ și $z = 3t + 6$

Deci $S = \{ (-11t - 9 ; t ; 3t + 6) \in \mathbb{R}^3 \mid t \in \mathbb{R} \}$

t) $\begin{cases} x + y + z = 9 \\ x + 2y + 3z = 14 \\ 3x + 2y + z = 22 \end{cases}$ $\begin{matrix} \cdot(-2) \\ \cdot(1) \end{matrix}$ $\begin{matrix} \cdot(-2) \\ \\ \cdot(1) \end{matrix}$

$\Rightarrow \begin{cases} -2x - 2y - 2z = -18 \\ x + 2y + 3z = 14 \end{cases}$

$-x + y = -4$

$\Rightarrow x = 4 + y$

$\begin{cases} -2x - 2y - 2z = -18 \\ 3x + 2y + z = 22 \end{cases}$

$x - z = 4$

$\Rightarrow x + y + z = 9 \Rightarrow 4 + z + y + z = 9$

$\Rightarrow y = 9 - 4 - 2z = -2z + 5$ Fie $z = t \Rightarrow x = 4 + t$ și $y = -2t + 5$

$\Rightarrow S = \{ (t + 4 ; -2t + 5 ; t) \in \mathbb{R}^3 \mid t \in \mathbb{R} \}$

4) $\begin{cases} 3x + 4y - z = -3 \\ 2x + y - z = -1 \\ x + 2y + z = 1 \\ x + y - 3z = -6 \end{cases}$ $\begin{matrix} \cdot(1) \\ \cdot(-1) \\ \\ \cdot(1) \end{matrix}$

$\Leftrightarrow$ $\begin{cases} 3x + 4y - z = -3 \\ -2x - y + z = 1 \end{cases}$ $\begin{cases} 3x + 4y - z = -3 \\ x + 2y + z = 1 \end{cases}$

$x + 3y = -2$ $\quad$ $4x + 6y = -2$

$\Leftrightarrow$ nouveau système : $\begin{cases} x + 3y = -2 \\ 4x + 6y = -2 \end{cases}$ $\begin{matrix} \cdot(-4) \\ \cdot(1) \end{matrix}$ $\Leftrightarrow$ $\begin{cases} -4x - 12y = 8 \\ 4x + 6y = -2 \end{cases}$

$-6y = 6$

$\Leftrightarrow y = -1$ $\quad \Leftrightarrow x = -2 - 3y = -2 - 3(-1)$

$\Leftrightarrow x = -2 + 3 = 1$ $\quad \Leftrightarrow x + 2y + z = 1$

$\Leftrightarrow z = 1 - x - 2y = 1 - 1 - 2(-1)$

$\Leftrightarrow z = 2$

* Vérification pour la 4e équation : $1 - 1 - 6 = -6$ ✓

$\Leftrightarrow \boxed{S = \{(1; -1; 2)\}}$

v) $\begin{cases} 2x + y + 3z = 3 \\ 3x - y + 4z = 2 \\ 4x + y - z = 5 \\ x + y + z = 4 \end{cases}$ $\begin{matrix} \cdot(1) \\ \cdot(1) \\ \\ \cdot(-1) \end{matrix}$

$\begin{cases} 2x + y + 3z = 3 \\ 3x - y + 4z = 2 \end{cases}$

$5x + 7z = 5$

$\begin{cases} 2x + y + 3z = 3 \\ -4x - y + z = -5 \end{cases}$

$-2x + 4z = -2$

$\Rightarrow$ nouveau système : $\begin{cases} 5x + 7z = 5 \\ -2x + 4z = -2 \end{cases}$ $\begin{matrix} \cdot(2) \\ \cdot(5) \end{matrix}$

$\Rightarrow \begin{cases} 10x + 14z = 10 \\ -10x + 20z = -10 \end{cases}$

$34z = 0$

$\Rightarrow z = 0 \Rightarrow x = 1 \Rightarrow y = 3 - 2 = 1$

* Vérification pour la 4e équation : $1 + 1 + 0 \neq 4$ non

$\Rightarrow \boxed{S = \emptyset}$

w) $\begin{cases} x + y + 2z = 4 \\ 3x - y + z = 3 \end{cases}$

$4x + 3z = 7 \Rightarrow 3z = 7 - 4x \Rightarrow y = 3x + \frac{7-4x}{3} - 3$

$\Rightarrow z = \frac{7-4x}{3}$

$\Rightarrow y = \frac{9x + 7 - 4x - 9}{3} = \frac{5x - 2}{3}$

Si $x = t \Rightarrow y = \frac{5t-2}{3}$ et $z = \frac{7-4t}{3}$

$\Rightarrow \boxed{S = \left\{ \left(t ; \frac{5t-2}{3} ; \frac{7-4t}{3} \right) \in \mathbb{R}^3 \mid t \in \mathbb{R} \right\}}$

7) $$\begin{cases} x+y+z+t+u=1 \\ x+2y+z+t+u=0 \\ x+y+3z+t+u=3 \\ x+y+z+4t+u=-2 \\ x+y+z+t+5u=5 \end{cases}$$

Écriture matricielle :

$$\left(\begin{array}{ccccc|c} 1&1&1&1&1&1 \\ 1&2&1&1&1&0 \\ 1&1&3&1&1&3 \\ 1&1&1&4&1&-2 \\ 1&1&1&1&5&5 \end{array}\right) \Rightarrow \left(\begin{array}{ccccc|c} 1&1&1&1&1&1 \\ 0&1&0&0&0&-1 \\ 0&0&2&0&0&2 \\ 0&0&0&3&0&-3 \\ 0&0&0&0&4&4 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccccc|c} 1&1&1&1&1&1 \\ 0&1&0&0&0&-1 \\ 0&0&1&0&0&1 \\ 0&0&0&1&0&-1 \\ 0&0&0&0&1&1 \end{array}\right) \Rightarrow \left(\begin{array}{ccccc|c} 1&0&0&0&0&1 \\ 0&1&0&0&0&-1 \\ 0&0&1&0&0&1 \\ 0&0&0&1&0&-1 \\ 0&0&0&0&1&1 \end{array}\right)$$

$$\Rightarrow S = \{(1;-1;1;-1;1)\}$$

2.5.22 Résoudre les systèmes suivants :

a) $\begin{cases} 2xy - 3y = 3 \\ y^2 - 4xy = -15 \end{cases}$

b) $\begin{cases} \frac{x}{y} - \frac{y}{x} = \frac{5}{6} \\ x + y = 30 \end{cases}$

c) $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{11}{10} \\ x + y = 11 \end{cases}$

d) $\begin{cases} x^2 + y^2 = 34 \\ xy - (x+y) = -13 \end{cases}$

e) $\begin{cases} \frac{1}{x^2} + \frac{1}{y^2} = 2 \\ xy + 1 = 0 \end{cases}$

f) $\begin{cases} x^2y^2 - xy = 30 \\ x^2 - y^2 = 5 \end{cases}$

g) $\begin{cases} x^2 + y^2 - 5xy = 15 \\ x + y + 3xy = -7 \end{cases}$

h) $\begin{cases} x + y = 9 \\ x^2 - y^2 = 9 \end{cases}$

a) $\begin{cases} 2xy - 3y = 3 \\ y^2 - 4xy = -15 \end{cases} \Leftrightarrow \begin{cases} 2xy = 3 + 3y \\ y^2 - 2(3+3y) = -15 \quad (*) \end{cases}$

$(*) \Leftrightarrow y^2 - 6 - 6y + 15 = 0 \Leftrightarrow y^2 - 6y + 9 = 0$

$\Leftrightarrow (y-3)^2 = 0 \Leftrightarrow y = 3 \Leftrightarrow x = \frac{9+3}{6} = 2$

$\Leftrightarrow S = \{(2;3)\}$

b) $\begin{cases} \frac{x}{y} - \frac{y}{x} = \frac{5}{6} \\ x + y = 30 \end{cases} \Leftrightarrow \begin{cases} \frac{x}{y} - \frac{y}{x} = \frac{5}{6} \\ y = 30 - x \end{cases}$

$\Leftrightarrow \frac{x}{30-x} - \frac{30-x}{x} = \frac{5}{6} \quad \Leftrightarrow ED = \mathbb{R}^* \setminus \{30\}$

$\Leftrightarrow 6x^2 - 6(30-x)^2 = 5x(30-x) \Leftrightarrow 6x^2 - 5400 + 360x - 6x^2 = 150x - 5x^2$

$\Leftrightarrow 5x^2 + 210x - 5400 = 0 \Leftrightarrow x^2 + 42x - 1080 = 0$

$\Leftrightarrow (x+60)(x-18) = 0 \Leftrightarrow x_1 = -60 \Leftrightarrow y_1 = 90$

$\Leftrightarrow x_2 = 18 \Leftrightarrow y_2 = 12$

$\Leftrightarrow S = \{(-66;90);(18;12)\}$

c) $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{11}{10} \\ x + y = 11 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{x} + \frac{1}{11-x} = \frac{11}{10} \\ y = 11 - x \end{cases}$ $(*) \Leftrightarrow$ EN $= \mathbb{R}^* \setminus \{11\}$

$(*) \quad 110 - 10x + 10x = 11x(11-x) \Leftrightarrow 11x^2 - 121x + 110 = 0$

$\Leftrightarrow x^2 - 11x + 10 = 0 \Leftrightarrow (x-10)(x-1) = 0 \Leftrightarrow x_1 = 1 \; , x_2 = 10$

$\Leftrightarrow y_1 = 10 \; , \; y_2 = 1 \Leftrightarrow$ $\boxed{S = \{(1;10);(10;1)\}}$

h) $\begin{cases} x + y = 9 \\ x^2 - y^2 = 9 \end{cases} \Leftrightarrow \begin{cases} x + y = 9 \\ (x-y)(x+y) = 9 \quad (*) \end{cases}$

$(*) \Leftrightarrow (x-y) \cdot 9 = 9 \Leftrightarrow x - y = 1$

nouveau système : $\begin{cases} x + y = 9 \\ x - y = 1 \end{cases}$

$2y = 8 \Leftrightarrow y = 4 \Leftrightarrow x = 5$

$\Leftrightarrow$ $\boxed{S = \{(5;4)\}}$