3.3.26 Résoudre les inéquations suivantes.

a) $\dfrac{x^2-4}{x^2-x} > 0$

b) $\dfrac{x\,(2\,x-3)^2}{x^2-4} < 0$

c) $\dfrac{4\,x^2+4\,x+1}{x^2-1} > 3$

d) $\dfrac{2}{x^2} \geq 1-x$

e) $\dfrac{x-3}{x^2-3\,x+2} > 0$

f) $\dfrac{3\,x^2-7\,x-20}{x^2+4\,x-12} \leq 0$

g) $\dfrac{x+1}{x-1} \leq \dfrac{x-1}{x+1}$

h) $\dfrac{13}{2\,x+1} \geq 9 - \dfrac{38}{4-x}$

i) $\dfrac{x-3}{-x^2+x-2} > 0$

j) $\dfrac{1}{x} \geq x$

k) $\dfrac{13}{2-x} \leq 7 - \dfrac{4}{3\,x+1}$

l) $\dfrac{12\,x^2-13\,x-14}{x-2} < 0$

m) $\dfrac{1}{x+1} + \dfrac{1}{x+2} < \dfrac{1}{x+3}$

n) $\dfrac{x}{3\,x-4} \geq \dfrac{1}{4}$

o) $\dfrac{1}{x+1} \leq \dfrac{x}{(x-2)(x+3)}$

p) $\dfrac{6}{4-x} - \dfrac{1}{1-x} \leq 1$

q) $1 \leq \dfrac{3}{2\,x-1} \leq 5$

r) $-2\,x \leq \dfrac{2\,x-1}{x} < 1$

g) $\dfrac{x+1}{x-1} \leq \dfrac{x-1}{x+1}$ $ED = \mathbb{R} \setminus \{\pm 1\}$

$\Rightarrow \dfrac{x+1}{x-1} - \dfrac{x-1}{x+1} \leq 0$

$\Rightarrow \dfrac{(x+1)^2 - (x-1)^2}{(x-1)(x+1)} \leq 0 \Rightarrow \dfrac{x^2+2x+1-(x^2-2x+1)}{(x-1)(x+1)} \leq 0$

$\Rightarrow \dfrac{\cancel{x^2}+2x+\cancel{1}-\cancel{x^2}+2x-\cancel{1}}{(x-1)(x+1)} \leq 0 \Rightarrow \dfrac{4x}{(x-1)(x+1)} \leq 0$

$-$ $\;-1\;$ $+$ $\;0\;$ $-$ $\;1\;$ $+$

Signe $f(x)$

$\boxed{S =]-\infty\,;-1[\, \cup \,[0\,;1[}$

m) $$\frac{1}{x+1}+\frac{1}{x+2}<\frac{1}{x+3} \qquad ED = \mathbb{R}\setminus\{-3\,;-2\,;-1\}$$

$$\Rightarrow \frac{1}{x+1}+\frac{1}{x+2}-\frac{1}{x+3}<0$$

$$\Rightarrow \frac{(x+2)(x+3)+(x+1)(x+3)-(x+1)(x+2)}{(x+1)(x+2)(x+3)}<0$$

$$\Rightarrow \frac{x^2+5x+6+x^2+4x+3-x^2-3x-2}{(x+1)(x+2)(x+3)}<0$$

$$\Rightarrow \frac{x^2+6x+7}{(x+1)(x+2)(x+3)}<0$$

alors x^2+6x+7 : $\Delta = 36-28 = 8$

$$\Rightarrow x_1 = \frac{-6-2\sqrt{2}}{2} = -3-\sqrt{2}$$

$$x_2 = \frac{-6+2\sqrt{2}}{2} = -3+\sqrt{2}$$

$\Rightarrow$

x	$-\infty$		$-3-\sqrt{2}$		-3		-2		$-3+\sqrt{2}$		-1		$+\infty$
x^2+6x+7		+	0	−	|	−	|	−	0	+	|	+	
$x+1$		−	|	−	|	−	|	−	|	−	0	+	
$(x+2)(x+3)$		+	|	+	0	−	0	+	|	+	|	+	
$\frac{x^2+6x+7}{(x+1)(x+2)(x+3)}$		−	0	+	||	−	||	+	0	−	||	+	

$$\boxed{S = \left]-\infty\,;-3-\sqrt{2}\right[\cup \left]-3\,;-2\right[\cup \left]-3+\sqrt{2}\,;-1\right[}$$

d) $\dfrac{1}{x+1} \leq \dfrac{x}{(x-2)(x+3)}$

$\Rightarrow \dfrac{1}{x+1} - \dfrac{x}{(x-2)(x+3)} \leq 0 \qquad ED_f = \mathbb{R} \setminus \{-3 ; -1 ; 2\}$

$\Rightarrow \dfrac{(x-2)(x+3) - x(x+1)}{(x+1)(x-2)(x+3)} \leq 0 \Rightarrow \dfrac{x^2+3x-2x-6-x^2-x}{(x+1)(x-2)(x+3)} \leq 0$

$\Rightarrow \dfrac{-6}{(x+1)(x-2)(x+3)} \leq 0$

Tableau des signes :

x	$-\infty$		-3		-1		2		$+\infty$
-6		$-$		$-$		$-$		$-$	
$x+1$		$-$		$-$	0	$+$		$+$	
$(x-2)(x+3)$		$+$	0	$-$		$-$	0	$+$	
$\dfrac{-6}{(x+1)(x-2)(x+3)}$		$+$	$\Vert$	$-$	$\Vert$	$+$	$\Vert$	$-$	

$\boxed{\Rightarrow S =]-3 ; -1[\cup]2 ; +\infty[}$

p) $\dfrac{6}{4-x} - \dfrac{1}{1-x} \leq 1$

$\Rightarrow \dfrac{6}{4-x} - \dfrac{1}{1-x} - 1 \leq 0$ $\qquad ED_f = \mathbb{R} \setminus \{1;4\}$

$\Rightarrow \dfrac{6(1-x) - (4-x) - 1(4-x)(1-x)}{(4-x)(1-x)} \leq 0$

$\Rightarrow \dfrac{6 - 6x - 4 + x - (4 - 4x - x + x^2)}{(4-x)(1-x)} \leq 0$

$\Rightarrow \dfrac{2 - 5x - x^2 + 5x - 4}{(4-x)(1-x)} \leq 0$

$\Rightarrow \dfrac{-x^2 - 2}{(4-x)(1-x)} \leq 0 \Rightarrow \dfrac{-(\overbrace{x^2+2}^{\text{toujour } > 0})}{(4-x)(1-x)} \leq 0$

Tableau des signes:

x	$-\infty$		1		4		$+\infty$
$-(x^2+2)$		$-$		$-$		$-$	
$(4-x)(1-x)$		$+$	0	$-$	0	$+$	
$\dfrac{-(x^2+2)}{(4-x)(1-x)}$		$-$	$\|\|$	$+$	$\|\|$	$-$	

$\boxed{S = \,]-\infty; 1[\, \cup \,]4; +\infty[}$

9) $1 \leq \dfrac{3}{2x-1} \leq 5$

* $1 \leq \dfrac{3}{2x-1} \Rightarrow 1 - \dfrac{3}{2x-1} \leq 0 \Rightarrow \dfrac{2x-1-3}{2x-1} \leq 0$

$\Rightarrow \dfrac{2x-4}{2x-1} \leq 0$

x		$\frac{1}{2}$		2	
Signe de $\frac{2x-4}{2x-1}$	$+$	$\|\|$	$-$	0	$+$

$\Rightarrow S_1 = \left] \frac{1}{2} ; 2 \right]$

* $\dfrac{3}{2x-1} \leq 5 \Rightarrow \dfrac{3}{2x-1} - 5 \leq 0$

$\Rightarrow \dfrac{3 - 5(2x-1)}{2x-1} \leq 0 \Rightarrow \dfrac{3 - 10x + 5}{2x-1} \leq 0$

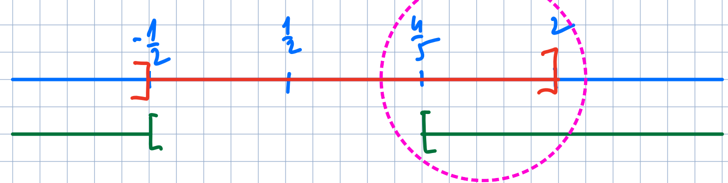
$\Rightarrow \dfrac{-10x+8}{2x-1} \leq 0$

x		$\frac{1}{2}$		$\frac{4}{5}$	
Signe de $\frac{-10x+8}{2x-1}$	$-$	$\|\|$	$+$	0	$-$

$-10x+8=0 \Rightarrow x = \dfrac{8}{10} = \dfrac{4}{5}$

$\Rightarrow S_2 = \left] -\infty ; -\frac{1}{2} \right[\cup \left[\frac{4}{5} ; +\infty \right[$

Donc $S = S_1 \cap S_2$



$\boxed{\Rightarrow S = \left[\frac{4}{5} ; 2 \right]}$

r) $-2x \le \frac{2x-1}{x} < 1$

* $-2x \le \frac{2x-1}{x} \Rightarrow -2x - \frac{2x-1}{x} \le 0$

$\Rightarrow \frac{-2x^2-2x+1}{x} \le 0 \quad \Big| \cdot (-1)$

$\Rightarrow \frac{2x^2+2x-1}{x} \ge 0$

On calcule les zéros de $2x^2+2x-1 \Rightarrow \Delta = 12 \Rightarrow \sqrt{\Delta} = \sqrt{4\cdot 3} = 2\sqrt{3}$

$\Rightarrow x_1 = \frac{-2-2\sqrt{3}}{4} = \frac{2(-1-\sqrt{3})}{4} = \frac{-1-\sqrt{3}}{2}$

$x_2 = \frac{-2+2\sqrt{3}}{4} = \frac{2(-1+\sqrt{3})}{4} = \frac{-1+\sqrt{3}}{2}$

x		$\frac{-1-\sqrt{3}}{2}$		0		$\frac{-1+\sqrt{3}}{2}$	
Signe de $\frac{2x^2+2x-1}{x}$	$-$	0	$+$	$\Vert$	$-$	0	$+$

$S_1 = \left[\frac{-1-\sqrt{3}}{2} ; 0\right[\cup \left[\frac{-1+\sqrt{3}}{2} ; +\infty\right[$

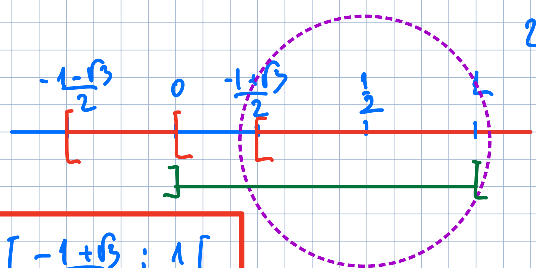
* $\frac{2x-1}{x} < 1 \Rightarrow \frac{2x-1}{x} - 1 < 0 \Rightarrow \frac{2x-1-x}{x} < 0$

$\Rightarrow \frac{x-1}{x} < 0$

x		0		1	
Signe de $\frac{x-1}{2x-1}$	$+$	$\Vert$	$-$	0	$+$

$\Rightarrow S_2 =]0 ; 1[$

Donc $S = S_1 \cap S_2$



$\boxed{\Rightarrow S = \left[\frac{-1+\sqrt{3}}{2} ; 1\right[}$