**CORRIGÉ** 1MS + 1MR

## 2.4 Fractions rationnelles

**2.4.1** Rendre les fractions rationnelles irréductibles :

a) $\dfrac{54a^3b^3}{15a^5b^2}$ b) $\dfrac{-16u^2v^2w^3}{-4u^3vw^2}$ c) $\dfrac{x-1}{2x-2}$

a) $\dfrac{54a^3b^3}{15a^5b^2} = \dfrac{18\,a^{3-5}\cdot b^{3-2}}{5} = \dfrac{18}{5}a^{-2}b = \boxed{\dfrac{18b}{5a^2}}$

b) $\dfrac{-16u^2v^2w^3}{-4u^3vw^2} = \dfrac{-16}{-4}u^{2-3}v^{2-1}w^{3-2} = 4u^{-1}vw = \boxed{\dfrac{4vw}{u}}$

c) $\dfrac{x-1}{2x-2} = \dfrac{\cancel{x-1}}{2(\cancel{x-1})} = \boxed{\dfrac{1}{2}}$

d) $\dfrac{2x-2y}{3y-3x}$ h) $\dfrac{3z^2-21z+36}{2z^2-12z+18}$ l) $\dfrac{6x^2+2x}{27x^3+1}$

e) $\dfrac{a^2-b^2}{(a-b)^2}$ i) $\dfrac{x^3-15x^2+75x-125}{x^2-25}$ m) $\dfrac{1-x^2+x^3-x^5}{x+x^2-x^3-x^4}$

f) $\dfrac{x^2-16}{x^2-5x+4}$ j) $\dfrac{x^4-y^4}{x^5-x^3y^2}$ n) $\dfrac{x^3+x^2-x-1}{x^3+2x^2-x-2}$

g) $\dfrac{x-x^3}{x^4+2x^3+x^2}$ k) $\dfrac{10x^2-10xy}{5x^2y^2-5x^4}$ o) $\dfrac{2x^3+9x^2+7x-6}{2x^3+x^2-13x+6}$

d) $\dfrac{2x-2y}{3y-3x} = \dfrac{2(\cancel{x-y})}{-3(\cancel{x-y})} = \boxed{-\dfrac{2}{3}}$

e) $\dfrac{a^2-b^2}{(a-b)^2} = \dfrac{(\cancel{a-b})(a+b)}{(\cancel{a-b})^2} = \boxed{\dfrac{a+b}{a-b}}$

f) $\dfrac{x^2-16}{x^2-5x+4} = \dfrac{(\cancel{x-4})(x+4)}{(x-1)(\cancel{x-4})} = \boxed{\dfrac{x+4}{x-1}}$

g) $\dfrac{x-x^3}{x^4+2x^3+x^2} = \dfrac{x(1-x^2)}{x^2(x^2+2x+1)} = \dfrac{x(1-x^2)}{x^2(x+1)^2} = \dfrac{\cancel{x}(\cancel{1+x})(1-x)}{\cancel{x^2}(\cancel{x+1})^2}$

$= \boxed{\dfrac{1-x}{x(x+1)}}$

h) $\dfrac{3z^2-21z+36}{2z^2-12z+18} = \dfrac{3(z-4)(z-3)}{2(z^2-6z+9)} = \dfrac{3(z-4)(\cancel{z-3})}{2(\cancel{z-3})^2} = \boxed{\dfrac{2(z-4)}{3(z-3)}}$

$a^3 - 3\cdot a^2 b + 3ab^2 - b^3 = (a-b)^3$

i) $\dfrac{x^3-15x^2+75x-125}{x^2-25} = \dfrac{x^3 - 3\cdot x^2\cdot 5 + 3\cdot x\cdot 25 - 5^3}{(x-5)(x+5)} = \dfrac{(\cancel{x-5})^3}{(\cancel{x-5})(x+5)}$

↑ id. remarquables

$= \boxed{\dfrac{(x-5)^2}{x+5}}$

j) $\dfrac{x^4-y^4}{x^5-x^3y^2} = \dfrac{(x^2)^2-(y^2)^2}{x^3(x^2-y^2)} = \dfrac{(\cancel{x^2-y^2})(x^2+y^2)}{x^3(\cancel{x^2-y^2})} = \boxed{\dfrac{x^2+y^2}{x^3}}$

k) $\dfrac{10x^2-10xy}{5x^2y^2-5x^4} = \dfrac{10x(x-y)}{5x^2(y^2-x^2)} = \dfrac{10x(x-y)}{5x^2(y-x)(y+x)}$

$= \dfrac{10x(\cancel{x-y})}{-5\cancel{x^2}(\cancel{x-y})(x+y)} = \boxed{\dfrac{2}{-x(x+y)}}$

l) $\dfrac{6x^2+2x}{27x^3+1} = \dfrac{2x(3x+1)}{\underbrace{(3x)^3+1^3}_{a^3+b^3=(a+b)(a^2-ab+b^2)}} = \dfrac{2x(\cancel{3x+1})}{(\cancel{3x+1})(9x^2-3x+1)} = \boxed{\dfrac{2x}{\underbrace{9x^2-3x+1}_{\Delta<0 \text{ pas facto!}}}}$

m) $$\frac{1-x^2+x^3-x^5}{x+x^2-x^3-x^4} = \frac{1+x^3-x^2-x^5}{x-x^3+x^2-x^4} = \frac{(1+x^3)-x^2(1+x^3)}{x(1-x^2)+x^2(1-x^2)}$$

$$= \frac{(1+x^3)(1-x^2)}{(1-x^2)(x+x^2)} \overset{a^3+b^3}{=} \frac{\cancel{(1+x)}(1^2-1\cdot x+x^2)\cancel{(1-x^2)}}{\cancel{(1-x^2)}\,x\,\cancel{(1+x)}}$$

$$= \boxed{\frac{1-x+x^2}{x}}$$

n) $$\frac{x^3+x^2-x-1}{x^3+2x^2-x-2} = \frac{x^2(x+1)-(x+1)}{x^2(x+2)-(x+2)} = \frac{(x+1)\cancel{(x^2-1)}}{(x+2)\cancel{(x^2-1)}}$$

$$= \boxed{\frac{x+1}{x+2}}$$

o) $$\frac{2x^3+9x^2+7x-6}{2x^3+x^2-13x+6}$$

* factoriser $2x^3+9x^2+7x-6$ par Horner

| | | | | |
|---|---|---|---|---|
| | 2 | 9 | 7 | -6 |
| -2 | | -4 | -10 | 6 |
| | 2 | 5 | -3 | $\boxed{0}$ = reste |

$$\Rightarrow 2x^3+9x^2+7x-6 = (x+2)(\underbrace{2x^2+5x-3}_{2(x+3)(x-\frac{1}{2})}) = (x+2)\,2\,(x+3)(x-\tfrac{1}{2})$$

$$= (x+2)\,2\,(x+3)\left(\frac{2x-1}{2}\right) = (x+2)(x+3)(2x-1)$$

* Factoriser $2x^3 + x^2 - 13x + 6$ par Horner

| | 2 | 1 | -13 | 6 |
|---|---|---|---|---|
| 2 | | +4 | 10 | -6 |
| | 2 | 5 | -3 | $\boxed{0}$ = Reste |

$$\Rightarrow \quad 2x^3 + x^2 - 13x + 6 = (x-2)\underbrace{(2x^2 + 5x - 3)}_{2(x+3)(x-\frac{1}{2})} = (x-2)(x+3)(2x-1)$$

D'où : $$\frac{2x^3 + 9x^2 + 7x - 6}{2x^3 + x^2 - 13x + 6} = \frac{(x+2)\cancel{(x+3)}\cancel{(2x-1)}}{(x-2)\cancel{(x+3)}\cancel{(2x-1)}} = \boxed{\frac{x+2}{x-2}}$$

**2.4.2** Effectuer et réduire :

a) $\dfrac{a+7}{a-1} \cdot \dfrac{a^2-1}{2a+14}$

b) $\dfrac{x+5}{7} \div \dfrac{2x+10}{x-8}$

c) $(x+y) \div \dfrac{x+y}{x-y}$

d) $\dfrac{z^2+z}{z-1} \cdot \dfrac{z-z^2}{z^3}$

e) $\dfrac{x+2}{2x-3} \div \dfrac{x^2-4}{2x^2-3x}$

f) $\dfrac{9x^2-4}{3x^2-5x+2} \cdot \dfrac{9x^4-6x^3+4x^2}{27x^4+8x}$

g) $\dfrac{x^2-6x+9}{x^2-1} \cdot \dfrac{2x-2}{x-3}$

h) $\dfrac{6x^2-5x-6}{x^2-4} \div \dfrac{2x^2-3x}{x+2}$

i) $\dfrac{5u^2+12u+4}{u^4-16} \cdot \dfrac{u^2-2u}{25u^2+20u+4}$

a) $$\frac{a+7}{a-1} \cdot \frac{a^2-1}{2a+14} = \frac{\cancel{(a+7)} \cdot \cancel{(a-1)}(a+1)}{\cancel{(a-1)} \quad 2\cancel{(a+7)}} = \boxed{\frac{a+1}{2}}$$

b) $$\frac{x+5}{7} \div \frac{2x+10}{x-8} = \frac{\cancel{(x+5)}}{7} \cdot \frac{(x-8)}{2\cancel{(x+5)}} = \boxed{\frac{x-8}{14}}$$

c) $$(x+y) \div \frac{x+y}{x-y} = \cancel{(x+y)} \cdot \frac{(x-y)}{\cancel{(x+y)}} = \boxed{x-y}$$

d) $$\frac{z^2+z}{z-1} \cdot \frac{z-z^2}{z^3} = \frac{z(z+1)}{z-1} \cdot \frac{z(1-z)}{z^3} = \frac{z^2(z+1)\left[-\cancel{(z-1)}\right]}{\cancel{(z-1)}\, z^3}$$

$$= -\frac{z^2(z+1)}{z^3} = \boxed{-\frac{z+1}{z}}$$

e) $$\frac{x+2}{2x-3} \div \frac{x^2-4}{2x^2-3x} = \frac{x+2}{2x-3} \cdot \frac{2x^2-3x}{x^2-4} = \frac{\cancel{x+2}}{\cancel{2x-3}} \cdot \frac{x\cancel{(2x-3)}}{(x-2)\cancel{(x+2)}}$$

$$= \boxed{\frac{x}{x-2}}$$

f) $\dfrac{9x^2-4}{3x^2-5x+2}\cdot\dfrac{9x^4-6x^3+4x^2}{27x^4+8x} = \dfrac{(3x)^2-2^2}{3(x-1)(x-\frac{2}{3})}\cdot\dfrac{x^2(9x^2-6x+4)}{x(27x^3+8)}$

$= \left(\dfrac{(\cancel{3x-2})(3x+2)}{(x-1)(\cancel{3x-2})}\right.\cdot\dfrac{\cancel{x^2}(9x^2-6x+4)}{\cancel{x}\,[\underbrace{(3x)^3+2^3}_{a^3+b^3}]}$

$= \dfrac{\cancel{3x+2}}{x-1}\cdot\dfrac{x(\cancel{9x^2-6x+4})}{(\cancel{3x+2})(\cancel{9x^2-6x+4})} = \boxed{\dfrac{x}{x-1}}$

g) $\dfrac{x^2-6x+9}{x^2-1}\cdot\dfrac{2x-2}{x-3} = \dfrac{(\cancel{x-3})^2}{(\cancel{x-1})(x+1)}\cdot\dfrac{2(\cancel{x-1})}{\cancel{x-3}} = \boxed{\dfrac{2(x-3)}{x+1}}$

h) $\dfrac{6x^2-5x-6}{x^2-4}\div\dfrac{2x^2-3x}{x+2} = \dfrac{6x^2-5x-6}{x^2-4}\cdot\dfrac{x+2}{2x^2-3x}$

$= \dfrac{6(x+\frac{2}{3})(x-\frac{3}{2})}{(x-2)(\cancel{x+2})}\cdot\dfrac{(\cancel{x+2})}{x(2x-3)} = \dfrac{\cancel{6}\,\left(\frac{3x+2}{\cancel{3}}\right)\left(\frac{2x-3}{\cancel{2}}\right)}{x(x-2)(2x-3)}$

$= \dfrac{(3x+2)(\cancel{2x-3})}{x(x-2)(\cancel{2x-3})} = \boxed{\dfrac{3x+2}{x(x-2)}}$

i) $\dfrac{5u^2+12u+4}{u^4-16}\cdot\dfrac{u^2-2u}{25u^2+20u+4} = \dfrac{5(u+2)(u+\frac{2}{5})}{(u^2-4)(u^2+4)}\cdot\dfrac{u(u-2)}{(5u+2)^2}$

$= \dfrac{\cancel{5}(\cancel{u+2})\left(\frac{\cancel{5u+2}}{\cancel{5}}\right)}{(\cancel{u-2})(\cancel{u+2})(u^2+4)}\cdot\dfrac{u(\cancel{u-2})}{(\cancel{5u+2})^2} = \boxed{\dfrac{u}{(5u+2)(u^2+4)}}$

**2.4.3** Effectuer et réduire :

a) $\dfrac{x}{x+3}+\dfrac{x+6}{x+3}$

b) $\dfrac{x}{x+3}-\dfrac{x+6}{x+3}$

c) $\dfrac{6}{x^2-4}-\dfrac{3x}{x^2-4}$

d) $\dfrac{2}{3x+1}+\dfrac{9}{(3x+1)^2}$

e) $\dfrac{5}{a}-\dfrac{2a-1}{a^2}+\dfrac{a+5}{a^3}$

f) $\dfrac{x}{x+1}+\dfrac{1}{x-1}-\dfrac{2x}{x^2-1}$

g) $\dfrac{x-3}{x+3}-\dfrac{2x}{x^2+5x+6}$

h) $\dfrac{1}{m}-\dfrac{m}{m^2-1}-\dfrac{2m+1}{m-m^3}$

i) $\dfrac{2y+1}{y^2+4y+4}-\dfrac{6y}{y^2-4}+\dfrac{3}{y-2}$

j) $\dfrac{13-5x}{6x^2-6}+\dfrac{3x}{x+1}-\dfrac{3x-5}{3x-3}$

a) $\dfrac{x}{x+3}+\dfrac{x+6}{x+3}=\dfrac{x+x+6}{x+3}=\dfrac{2x+6}{x+3}=\dfrac{2\cancel{(x+3)}}{\cancel{x+3}}=\boxed{2}$

même dénominateur

b) $\dfrac{x}{x+3}-\dfrac{x+6}{x+3}=\dfrac{x-x-6}{x+3}=\boxed{\dfrac{-6}{x+3}}$

attention signe (−) => changer de signe

c) $\dfrac{6}{x^2-4}-\dfrac{3x}{x^2-4}=\dfrac{6-3x}{x^2-4}=\dfrac{3(2-x)}{(x-2)(x+2)}=\dfrac{-3\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$=\boxed{\dfrac{-3}{x+2}}$

d) $\dfrac{2}{3x+1}+\dfrac{9}{(3x+1)^2}=\dfrac{2(3x+1)+9}{(3x+1)^2}=\dfrac{6x+2+9}{(3x+1)^2}=\boxed{\dfrac{6x+11}{(3x+1)^2}}$

e) $$\frac{5}{a} - \frac{2a-1}{a^2} + \frac{a+5}{a^3} = \frac{5a^2 - a(2a-1) + a+5}{a^3}$$

$$= \frac{5a^2 - 2a^2 + a + a + 5}{a^3} = \boxed{\frac{3a^2 + 2a + 5}{a^3}}$$

f) $$\frac{x}{x+1} + \frac{1}{x-1} - \frac{2x}{\underbrace{x^2-1}_{(x-1)(x+1)}} = \frac{x(x-1) + (x+1) - 2x}{(x-1)(x+1)}$$

$$= \frac{x^2 \cancel{-x} \cancel{+x} + 1 - 2x}{(x-1)(x+1)} = \frac{x^2 - 2x + 1}{(x-1)(x+1)} = \frac{(\cancel{x-1})^2}{(\cancel{x-1})(x+1)} = \frac{x-1}{x+1}$$

g) $$\frac{x-3}{x+3} - \frac{2x}{\underbrace{x^2+5x+6}_{(x+3)(x+2)}} = \frac{(x-3)(x+2) - 2x}{(x+3)(x+2)} = \frac{x^2+2x-3x-6-2x}{(x+3)(x+2)}$$

$$= \boxed{\frac{x^2-3x-6}{(x+2)(x+3)}}$$

h) $$\frac{1}{m} - \frac{m}{m^2-1} - \frac{2m+1}{m-m^3} = \frac{1}{m} - \frac{m}{(m-1)(m+1)} - \frac{2m+1}{m(1-m^2)}$$

$$= \frac{1}{m} - \frac{m}{(m-1)(m+1)} - \frac{2m+1}{m(1-m)(1+m)} = \frac{1}{m} - \frac{m}{(m-1)(m+1)} - \frac{2m+1}{m(-(m-1))(m+1)}$$

$$= \frac{1}{m} - \frac{m}{(m-1)(m+1)} + \frac{2m+1}{m(m-1)(m+1)} = \frac{1(m-1)(m+1) - m\cdot m + 2m+1}{m(m-1)(m+1)}$$

$$= \frac{m^2-1 - m^2 + 2m + 1}{m(m-1)(m+1)} = \frac{2\cancel{m}}{\cancel{m}(m-1)(m+1)} = \boxed{\frac{2}{(m-1)(m+1)}}$$

i) $$\frac{2y+1}{y^2+4y+4} - \frac{6y}{y^2-4} + \frac{3}{y-2} = \frac{2y+1}{(y+2)^2} - \frac{6y}{(y-2)(y+2)} + \frac{3}{y-2}$$

$\Rightarrow$ dénominateur commun : $(y+2)^2 \cdot (y-2)$

$$\Rightarrow \frac{(2y+1)(y-2) - 6y(y+2) + 3(y+2)^2}{(y+2)^2(y-2)}$$

$$= \frac{2y^2 - 4y + y - 2 - 6y^2 - 12y + 3(y^2+4y+4)}{(y+2)^2(y-2)}$$

$$= \frac{-4y^2 - 15y - 2 + 3y^2 + 12y + 12}{(y+2)^2(y-2)} = \frac{-y^2 - 3y + 10}{(y+2)^2(y-2)}$$

$$= \frac{-(y^2+3y-10)}{(y+2)^2(y-2)} = \frac{-(y+5)\cancel{(y-2)}}{(y+2)^2\cancel{(y-2)}} = \boxed{-\frac{y+5}{(y+2)^2}}$$

j) $$\frac{13-5x}{6x^2-6} + \frac{3x}{x+1} - \frac{3x-5}{3x-3} = \frac{13-5x}{6(x^2-1)} + \frac{3x}{x+1} - \frac{3x-5}{3(x-1)}$$

$$= \frac{13-5x}{6(x-1)(x+1)} + \frac{3x}{x+1} - \frac{3x-5}{3(x-1)}$$

Dénominateur commun : $6(x-1)(x+1)$

$$\Rightarrow \frac{13-5x + 3x \cdot 6(x-1) - 2(3x-5)(x+1)}{6(x-1)(x+1)}$$

$$= \frac{13-5x+18x^2-18x-2(3x^2+3x-5x-5)}{6(x-1)(x+1)} = \frac{13-23x+18x^2-2(3x^2-2x-5)}{6(x-1)(x+1)}$$

$$= \frac{13-23x+18x^2-6x^2+4x+10}{6(x-1)(x+1)} = \boxed{\frac{12x^2-19x+23}{6(x-1)(x+1)}}$$

**2.4.4** Effectuer et réduire :

a) $\dfrac{x}{x+y}+\dfrac{y}{x-y}-\dfrac{2\,xy}{x^2-y^2}$

b) $\dfrac{x+y}{x-y}+\dfrac{x-2\,y}{x+y}+\dfrac{x^2+3\,y^2}{y^2-x^2}$

c) $\dfrac{1}{x}-\dfrac{x}{x^2-1}-\dfrac{2\,x+1}{x-x^3}$

d) $\dfrac{4}{x^2-y^2}+\dfrac{3\,y}{x^2y-x^3}-\dfrac{x-3\,y}{x^3-xy^2}$

e) $\dfrac{x^3-y^3}{x^2-y^2}-\dfrac{x^2y+xy^2}{x^2+xy}$

f) $\dfrac{x-3}{x+3}-\dfrac{4\,x-6\,y}{xy+3\,y+2\,x+6}+\dfrac{y+6}{y+2}$

g) $\dfrac{x+2}{x^2+7\,x+10}-\dfrac{x-3}{x^2-8\,x+15}+\dfrac{x^2-15}{x^2-25}$

h) $\dfrac{2\,x^2-x-7}{x^4-5\,x^2+4}-\dfrac{x-3}{x^3-2\,x^2-x+2}$

a) $\dfrac{x}{x+y}+\dfrac{y}{x-y}-\dfrac{2xy}{x^2-y^2}=\dfrac{x}{x+y}+\dfrac{y}{x-y}-\dfrac{2xy}{(x-y)(x+y)}$

$=\dfrac{x(x-y)+y(x+y)-2xy}{(x-y)(x+y)}=\dfrac{x^2-\cancel{xy}+\cancel{yx}+y^2-2xy}{(x-y)(x+y)}$

$=\dfrac{x^2-2xy+y^2}{(x-y)(x+y)}=\dfrac{(\cancel{x-y})^2}{(\cancel{x-y})(x+y)}=\boxed{\dfrac{x-y}{x+y}}$

b) $\dfrac{x+y}{x-y}+\dfrac{x-2y}{x+y}+\dfrac{x^2+3y^2}{y^2-x^2}=\dfrac{x+y}{x-y}+\dfrac{x-2y}{x+y}+\dfrac{x^2+3y^2}{-(x^2-y^2)}$

$\Rightarrow \dfrac{x+y}{x-y}+\dfrac{x-2y}{x+y}-\dfrac{x^2+3y^2}{(x-y)(x+y)}=\dfrac{(x+y)(x+y)+(x-y)(x-2y)-(x^2+3y^2)}{(x-y)(x+y)}$

$$\Rightarrow \frac{(x+y)^2 + \cancel{x^2} - 2xy - yx + 2y^2 - \cancel{x^2} - 3y^2}{(x-y)(x+y)} = \frac{x^2 + 2xy + \cancel{y^2} - 3xy - \cancel{y^2}}{(x-y)(x+y)}$$

$$\Rightarrow \frac{x^2 - xy}{(x-y)(x+y)} = \frac{x\cancel{(x-y)}}{\cancel{(x-y)}(x+y)} = \boxed{\frac{x}{x+y}}$$

c) $$\frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{x-x^3} = \frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{x(1-x^2)}$$

$$\Rightarrow \frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{-x(x^2-1)} = \frac{1}{x} - \frac{x}{x^2-1} + \frac{2x+1}{x(x^2-1)}$$

Dénominateur commun

$$\Rightarrow \frac{1(x^2-1) - x.x + 2x+1}{x(x^2-1)} = \frac{\cancel{x^2} - \cancel{1} - \cancel{x^2} + 2x + \cancel{1}}{x(x^2-1)} = \frac{2\cancel{x}}{\cancel{x}(x^2-1)}$$

$$= \boxed{\frac{2}{x^2-1}}$$

d) $$\frac{4}{x^2-y^2} + \frac{3y}{x^2y - x^3} - \frac{x-3y}{x^3 - xy^2} = \frac{4}{x^2-y^2} + \frac{3y}{x^2(y-x)} - \frac{x-3y}{x(x^2-y^2)}$$

$$\Rightarrow \frac{4}{x^2-y^2} + \frac{3y}{-x^2(x-y)} - \frac{x-3y}{x(x^2-y^2)} = \frac{4}{x^2-y^2} - \frac{3y}{x^2(x-y)} - \frac{x-3y}{x(x^2-y^2)}$$

$$\Rightarrow \frac{4}{(x-y)(x+y)} - \frac{3y}{x^2(x-y)} - \frac{x-3y}{x(x-y)(x+y)}$$

$\rightarrow$ Dénominateur commun : $x^2(x-y)(x+y)$

$$\Rightarrow \frac{4x^2 - 3y(x+y) - x(x-3y)}{x^2(x-y)(x+y)} = \frac{4x^2 - \cancel{3yx} - 3y^2 - x^2 + \cancel{3xy}}{x^2(x-y)(x+y)}$$

$$= \frac{3x^2 - 3y^2}{x^2(x-y)(x+y)} = \frac{3\cancel{(x^2-y^2)}}{x^2 \underbrace{\cancel{(x-y)(x+y)}}_{= x^2 - y^2}} = \boxed{\frac{3}{x^2}}$$

e) $$\frac{x^3 - y^3}{x^2 - y^2} - \frac{x^2y + xy^2}{x^2 + xy} = \frac{\cancel{(x-y)}(x^2 + xy + y^2)}{\cancel{(x-y)}(x+y)} - \frac{\cancel{x}y\cancel{(x+y)}}{\cancel{x}\cancel{(x+y)}}$$

$$\Rightarrow \frac{x^2 + xy + y^2}{x+y} - y = \frac{x^2 + xy + y^2 - y(x+y)}{x+y}$$

$$= \frac{x^2 + \cancel{xy} + \cancel{y^2} - \cancel{yx} - \cancel{y^2}}{x+y} = \boxed{\frac{x^2}{x+y}}$$

f) $$\frac{x-3}{x+3} - \frac{4x-6y}{xy+3y+2x+6} + \frac{y+6}{y+2}$$

$$= \frac{x-3}{x+3} - \frac{4x-6y}{y(x+3)+2(x+3)} + \frac{y+6}{y+2} = \frac{x-3}{x+3} - \frac{4x-6y}{(x+3)(y+2)} + \frac{y+6}{y+2}$$

$\Rightarrow$ Dénominateur commun : $(x+3)(y+2)$

$$= \frac{(x-3)(y+2) - (4x-6y) + (x+3)(y+6)}{(x+3)(y+2)}$$

$$= \frac{xy + 2x - 3y - 6 - 4x + 6y + xy + 6x + 3y + 18}{(x+3)(y+2)} = \frac{2xy + 4x + 6y + 12}{(x+3)(y+2)}$$

$$= \frac{2x(y+2) + 6(y+2)}{(x+3)(y+2)} = \frac{(y+2)(2x+6)}{(x+3)(y+2)} = \frac{\cancel{(y+2)} \cdot 2 \cdot \cancel{(x+3)}}{\cancel{(x+3)}\cancel{(y+2)}} = \boxed{2}$$

g) $\dfrac{x+2}{x^2+7x+10} - \dfrac{x-3}{x^2-8x+15} + \dfrac{x^2-15}{x^2-25} = \dfrac{\cancel{(x+2)}}{\cancel{(x+2)}(x+5)} - \dfrac{\cancel{(x-3)}}{\cancel{(x-3)}(x-5)} + \dfrac{x^2-15}{(x-5)(x+5)}$

$= \dfrac{1}{x+5} - \dfrac{1}{x-5} + \dfrac{x^2-15}{(x-5)(x+5)} = \dfrac{x-5-(x+5)+x^2-15}{(x-5)(x+5)}$

$= \dfrac{\cancel{x}-5-\cancel{x}-5+x^2-15}{(x-5)(x+5)} = \dfrac{x^2-25}{\underbrace{(x-5)(x+5)}_{x^2-25}} = \boxed{1}$

h) $\dfrac{2x^2-x-7}{\underbrace{x^4-5x^2+4}_{\text{Trinôme unitaire}}} - \dfrac{x-3}{x^3-2x^2-x+2}$

* $x^4-5x^2+4 = (x^2-1)(x^2-4) = (x-1)(x+1)(x-2)(x+2)$

* $x^3-2x^2-x+2 = x^2(x-2)-(x-2) = (x-2)(x^2-1) = (x-2)(x-1)(x+1)$

$= \dfrac{2x^2-x-7}{\underbrace{(x-1)(x+1)(x-2)(x+2)}_{\text{Dénominateur commun}}} - \dfrac{x-3}{(x-1)(x+1)(x-2)} = \dfrac{2x^2-x-7-(x-3)(x+2)}{(x-1)(x+1)(x-2)(x+2)}$

$= \dfrac{2x^2-x-7-(x^2+\overbrace{2x-3x}^{-x}-6)}{(x-1)(x+1)(x-2)(x+2)} = \dfrac{2x^2-\cancel{x}-7-x^2+\cancel{x}+6}{(x-1)(x+1)(x-2)(x+2)}$

$= \dfrac{\cancel{x^2-1}}{\underbrace{\cancel{(x-1)(x+1)}}_{x^2-1}(x-2)(x+2)} = \dfrac{1}{(x-2)(x+2)} = \boxed{\dfrac{1}{x^2-4}}$

**2.4.5** Effectuer et réduire :

a) $\left(\dfrac{z+2}{z}-\dfrac{2}{z^2+z}\right)\left(\dfrac{1}{z}+1\right)$

b) $\left[\left(x+\dfrac{2x}{x-2}\right)\left(\dfrac{2x}{x-2}-2\right)\right]\div\dfrac{4x^2}{x^2-4}$

c) $\left(\dfrac{1}{u}-\dfrac{2}{u^2}-\dfrac{3}{u^3}\right)\div\left(\dfrac{1}{u^2}-1\right)$

d) $\dfrac{2x-1}{x+y}\cdot\dfrac{x^2-y^2}{4x^2-1}\cdot\left(1-\dfrac{2x}{2x-1}+\dfrac{1}{2x+1}\right)$

a) $$\left(\frac{z+2}{z}-\frac{2}{z^2+z}\right)\left(\frac{1}{z}+1\right)=\left(\frac{z+2}{z}-\frac{2}{z(z+1)}\right)\left(\frac{1+z}{z}\right)$$

$$\Rightarrow \frac{(z+2)(z+1)-2}{z(z+1)}\cdot\frac{1+z}{z}=\frac{(z^2+z+2z+2-2)\cancel{(1+z)}}{z^2\cancel{(z+1)}}$$

$$\Rightarrow \frac{z^2+3z}{z^2}=\frac{\cancel{z}(z+3)}{\cancel{z}^2}=\boxed{\frac{z+3}{z}}$$

b) $$\left[\left(x+\frac{2x}{x-2}\right)\left(\frac{2x}{x-2}-2\right)\right]\div\frac{4x^2}{x^2-4}$$

$$=\left(\frac{x(x-2)+2x}{x-2}\right)\left(\frac{2x-2(x-2)}{x-2}\right)\cdot\frac{(x-2)(x+2)}{4x^2}$$

$$=\frac{x^2-2x+2x}{x-2}\cdot\left(\frac{2x-2x+4}{x-2}\right)\cdot\frac{(x-2)(x+2)}{4x^2}$$

$$=\frac{\cancel{x^2\cdot 4}}{(\cancel{x-2})^2}\cdot\frac{(\cancel{x-2})(x+2)}{\cancel{4x^2}}=\boxed{\frac{x+2}{x-2}}$$

c) $$\left(\frac{1}{u} - \frac{2}{u^2} - \frac{3}{u^3}\right) \div \left(\frac{1}{u^2} - 1\right) = \left(\frac{u^2 - 2u - 3}{u^3}\right) \div \left(\frac{1-u^2}{u^2}\right)$$

$$\Rightarrow \frac{u^2 - 2u - 3}{\cancel{u^3}} \cdot \frac{\cancel{u^2}}{1-u^2} = \frac{(u-3)\cancel{(u+1)}}{u(1-u)\cancel{(1+u)}} = \boxed{\frac{u-3}{u(1-u)}}$$

d) $$\frac{2x-1}{x+y} \cdot \frac{x^2-y^2}{4x^2-1} \cdot \left(1 - \frac{2x}{2x-1} + \frac{1}{2x+1}\right)$$

$$\Rightarrow \frac{\cancel{(2x-1)}(x-y)\cancel{(x+y)}}{\cancel{(x+y)}\cancel{(2x-1)}(2x+1)} \cdot \left(\frac{(2x-1)(2x+1) - 2x(2x+1) + (2x-1)}{(2x-1)(2x+1)}\right)$$

$$\Rightarrow \frac{x-y}{2x+1} \cdot \frac{\cancel{4x^2} - 1 - \cancel{4x^2} - \cancel{2x} + \cancel{2x} - 1}{(2x-1)(2x+1)} = \boxed{-\frac{2(x-y)}{(2x-1)(2x+1)^2}}$$