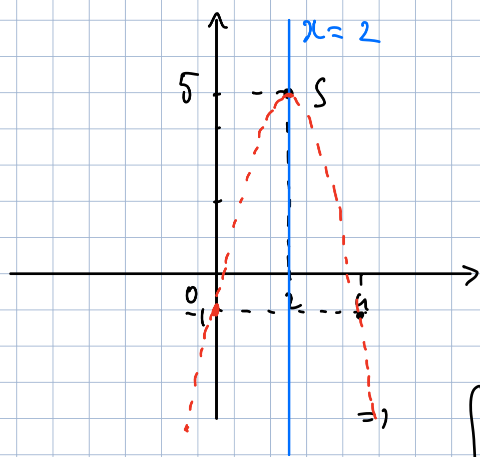
Ex 3.3.16

a) $S(2;5)$ et le graphe passe par $A(4;-1)$



$f(x) = ax^2 + bx + c$

$$\begin{cases} -\frac{b}{2a} = 2 & \text{: axe de symétrie} \\ f(4) = -1 \\ f(2) = 5 \end{cases}$$

$$\begin{cases} -\frac{b}{2a} = 2 \iff f(0) = -1 \iff c = -1 \\ 16a + 4b + c = -1 \\ 4a + 2b + c = 5 \end{cases}$$

$$\begin{cases} \boxed{c = -1} \\ 16a + 4b + c = -1 \\ 4a + 2b + c = 5 \end{cases} \Rightarrow \begin{cases} 16a + 4b - 1 = -1 \\ 4a + 2b - 1 = 5 \end{cases}$$

$$\Rightarrow \begin{cases} 16a + 4b = 0 & \Rightarrow 4b = -16a \Rightarrow b = -4a \\ 4a + 2b = 6 & \Rightarrow 4a + 2(-4a) = 6 \end{cases}$$

$\Rightarrow 4a - 8a = 6 \Rightarrow -4a = 6$

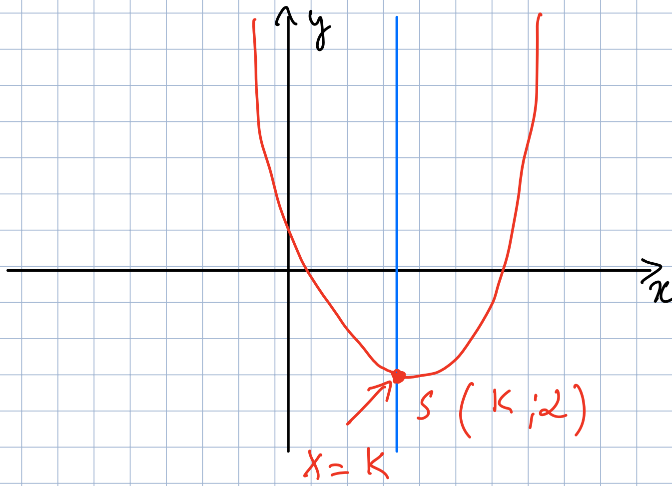
$$\boxed{a = -\frac{3}{2}}$$

$b = -4a = -4\left(-\frac{3}{2}\right) = \frac{12}{2} = \boxed{6 = b}$

Donc $\boxed{f(x) = -\frac{3}{2}x^2 + 6x - 1}$

Méthode 2 :

$$\boxed{f(x) = a(x-k)^2 + \alpha}$$



Dans notre cas $S(2;5)$

$$\Rightarrow f(x) = a(x-k)^2 + \alpha$$

$$f(x) = a(x-2)^2 + 5$$

$f(x)$ passe par $(4;-1) \Rightarrow f(4) = -1$

$$\Rightarrow a(4-2)^2 + 5 = -1$$

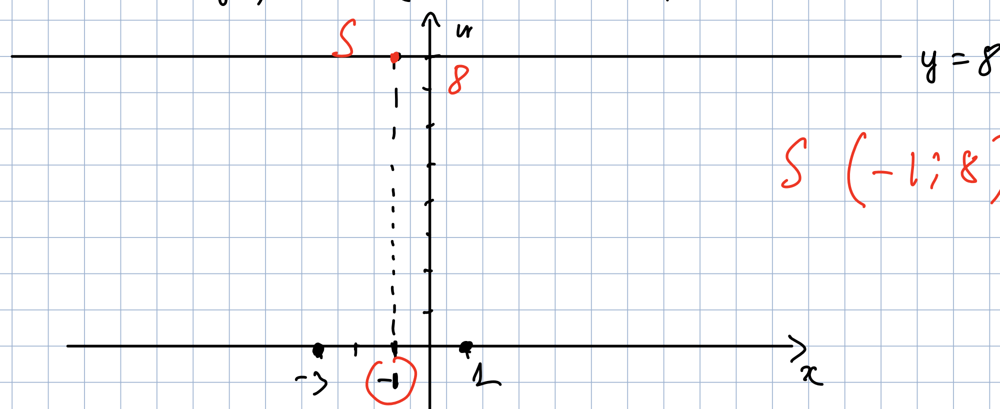
$$\Rightarrow a = -\frac{3}{2}$$

$$\Rightarrow f(x) = -\frac{3}{2}(x-2)^2 + 5 = \ldots = \boxed{-\frac{3}{2}x^2 + 6x - 1}$$

b) $f(x)$ coupe Ox en $x = -3$ et $x = 1$

$$\Rightarrow f(x) = a(x - x_1)(x - x_2)$$

$$f(x) = a(x - 1)(x + 3)$$



$S(-1; 8)$

$$f(x) = a(x-1)(x+3)$$

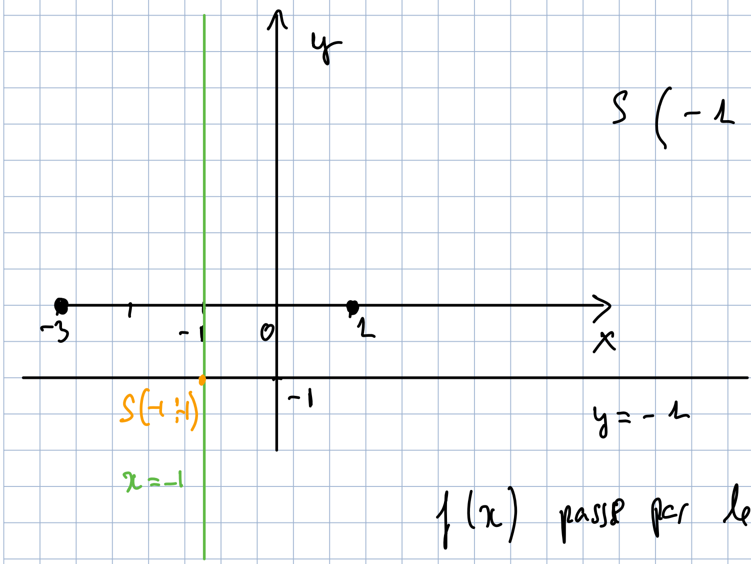
$$\Rightarrow f(-1) = 8 \Rightarrow 8 = a(-1-1)(-1+3)$$

$$8 = a(-2)(2) = -4a$$

$$a = -2$$

$$\Rightarrow f(x) = -2(x-1)(x+3) = \underline{-2x^2 - 4x + 6}$$

c) $f(x)$ coupe l'axe Ox en $x=-3$ et $x=1$ et qui est tangente à la droite $y=-1$



$S(-1;-1)$

$f(x) = a(x-x_1)(x-x_2)$

$f(x) = a(x-1)(x+3)$

$f(x)$ passe par le sommet $S(-1;-1)$

$\Rightarrow f(-1) = -1$

$$a(-1-1)(-1+3) = -1$$
$$a(-2)(2) = -1$$
$$-4a = -1$$
$$a = \frac{1}{4}$$

$\Rightarrow \boxed{f(x) = \frac{1}{4}(x-1)(x+3)}$

$$= \frac{1}{4}\left(x^2+3x-x-3\right)$$
$$= \frac{1}{4}\left(x^2+2x-3\right)$$

$\Rightarrow \boxed{f(x) = \frac{1}{4}x^2+\frac{1}{2}x-\frac{3}{4}}$

Ex 3.3.18 :

Déterminer les points d'intersection des graphes de f et de g.

a) $f(x) = x^2 - 5x + 4$ et $g(x) = -4x + 10$

$\Rightarrow$ points d'intersection $\Leftrightarrow$ résoudre $f(x) = g(x)$

alors $x^2 - 5x + 4 = -4x + 10$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = -2 \Rightarrow g(-2) = -4\cdot(-2) + 10 = 8 + 10 = 18$

$\Rightarrow I_1\ (-2\ ;\ 18)$

$\Rightarrow x = 3 \Rightarrow g(3) = -4\cdot 3 + 10 = -12 + 10 = -2$

$\Rightarrow I_2\ (3\ ;\ -2)$

Donc deux points d'intersection : $\boxed{I_1\ (-2\ ;\ 18) \text{ et } I_2\ (3\ ;\ -2)}$

b) $f(x) = x^2 + x$ et $g(x) = 2x^2 - 6$

$\Rightarrow f(x) = g(x) \Leftrightarrow x^2 + x = 2x^2 - 6$

$\Leftrightarrow x^2 - x - 6 = 0$

$\Leftrightarrow (x+2)(x-3) = 0$

- $x = -2 \Rightarrow f(-2) = (-2)^2 + (-2) = 4 - 2 = 2$

  $\Rightarrow I_1\ (-2\ ;\ 2)$

- $x = 3 \Rightarrow f(3) = (3)^2 + 3 = 12 \Rightarrow I_2\ (3\ ;\ 12)$

Donc deux points d'intersection : $\boxed{I_1\ (-2\ ;\ 2) \text{ et } I_2\ (3\ ;\ 12)}$

e) $1 - 3x \leq \frac{1}{3}x + 2$

$\Leftrightarrow -3x - \frac{1}{3}x \leq -1 + 2 \Leftrightarrow -9x - x \leq -3 + 6$

$\Leftrightarrow -10x \leq 3 \Leftrightarrow x \geq -\frac{3}{10}$

$\Rightarrow \boxed{S = \left[\frac{-3}{10} ; +\infty \right[}$

f) $3(1-x) > \frac{2}{5}x$

$\Leftrightarrow 3 \cdot 5(1-x) > 2x \Leftrightarrow 15 - 15x > 2x$

$\Leftrightarrow -17x > -15 \Leftrightarrow x < \frac{15}{17}$

$\Rightarrow \boxed{S = \left] -\infty ; \frac{15}{17} \right[}$