2.2 Factoriser une expression

2.2.1 Factoriser :

a) $xy + y = y(x+1)$

b) $ma + ap = a(m+p)$

c) $a^3x^2 - a^2x^3 = a^2x^2(a-x)$

d) $4uv - 2uw = 2u(2v-w)$

e) $6a^2 + 4ab = 2a(3a+2b)$

f) $24y^3z^5 - 36yz^2 = 12yz^2(2y^2z^3-3)$

g) $2yz^5 + 8y^2z^4 + 6y^3z^3 - 2y^4z^2$

h) $15m^7n^2 - 10m^5n^3$

i) $3a^2bc^2 - abc^3 = abc^2(3a-c)$

j) $(2a+3b)(2x+y) + (3a+5b)(2x+y)$

k) $3ab^4c^3 - ab^3c^2 = ab^3c^2(3bc-1)$

l) $2u^3v^2 + 8u^3v^3 - 6u^4v$

m) $(x-3)(x+1) + 2(x-3)^2 - (x-3)$

n) $(u+v)^3 - (u+v)^2$

o) $2a(a-b) - (a-b)^2$

g) $2yz^5 + 8y^2z^4 + 6y^3z^3 - 2y^4z^2 = \boxed{2yz^2(z^3 + 4yz^2 + 3y^2z - y^3)}$

h) $15m^7n^2 - 10m^5n^3 = \boxed{5m^5n^2(3m^2 - 2n)}$

j) $(2a+3b)(2x+y) + (3a+5b)(2x+y) = (2x+y)(2a+3b+3a+5b)$

$= \boxed{(2x+y)(5a+8b)}$

l) $2u^3v^2 + 8u^3v^3 - 6u^4v = \boxed{2u^3v(v + 4v^2 - 3u)}$

m) $(x-3)(x+1) + 2(x-3)^2 - (x-3) = (x-3)(x+1+2(x-3)-1)$

$= (x-3)(x+1+2x-6-1) = (x-3)(3x-6) = \boxed{3(x-3)(x-2)}$

n) $(u+v)^3 - (u+v)^2 = \boxed{(u+v)^2(u+v-1)}$

o) $2a(a-b) - (a-b)^2 = (a-b)(2a-(a-b)) = \boxed{(a-b)(a+b)}$

2.2.2 Factoriser :

a) $a^2b^2 - m^2 = (ab-m)(ab+m)$

b) $x^4 - y^2 = (x^2-y)(x^2+y)$

c) $a^2 - \dfrac{1}{16} = \left(a - \frac{1}{4}\right)\left(a + \frac{1}{4}\right)$

d) $(a+b)^2 - x^2 = (a+b-x)(a+b+x)$

e) $(ax+2y)^2 - (2x-3y)^2$

f) $(a-b)^2 - 1 = (a-b+1)(a-b-1)$

g) $3a^2 - 3 = 3(a^2-1) = 3(a-1)(a+1)$

h) $4x^5y^2 - 9x^3$

i) $a^4 - b^4 = (a^2-b^2)(a^2+b^2)$
$= (a-b)(a+b)(a^2+b^2)$

j) $a^5 - a$

k) $\dfrac{u^4}{625} - \dfrac{v^4}{81}$

l) $x^5y^4 - x$

m) $a^2 + 2a + 1 = (a+1)^2$

n) $1 + 2x^2 + x^4 = (1+x^2)^2$

o) $a^4 + 9b^2 - 6a^2b = (a^2-3b)^2$

p) $9x^4 + 16y^2 + 24x^2y = (3x^2+4y)^2$

q) $x^2 - x + \dfrac{1}{4}$

r) $\dfrac{xy}{3} + \dfrac{y^2}{9} + \dfrac{x^2}{4}$

s) $(a+b)^2 - 2(a+b)c + c^2$

t) $5x^2 - 10x + 5 = 5(x^2-2x+1) = 5(x-1)^2$

u) $x^2(a+b) + 2(a+b)x + a + b$

e) $(ax+2y)^2 - (2x-3y)^2 = (ax+2y-2x+3y)(ax+2y+2x-3y)$
$= \underline{(ax-2x+5y)(ax+2x-y)}$

h) $4x^5y^2 - 9x^3 = x^3(\underbrace{4x^2y^2-9}_{a^2-b^2}) = \underline{x^3(2xy-3)(2xy+3)}$

j) $a^5 - a = a(a^4-1) = a(a^2-1)(a^2+1) = \underline{a(a-1)(a+1)(a^2+1)}$

k) $\dfrac{u^4}{625} - \dfrac{v^4}{81} = \left(\dfrac{u^2}{25}\right)^2 - \left(\dfrac{v^2}{9}\right)^2 = \left(\dfrac{u^2}{25} - \dfrac{v^2}{9}\right)\left(\dfrac{u^2}{25} + \dfrac{v^2}{9}\right)$
$= \underline{\left(\dfrac{u}{5} - \dfrac{v}{3}\right)\left(\dfrac{u}{5} + \dfrac{v}{3}\right)\left(\dfrac{u^2}{25} + \dfrac{v^2}{9}\right)}$

l) $x^5y^4 - x = x(x^4y^4 - 1) = x(x^2y^2 - 1)(x^2y^2 + 1)$

$= \underline{x(x^2y^2 + 1)(xy - 1)(xy + 1)}$

q) $\underbrace{x^2 - x + \frac{1}{4}}_{(a-b)^2} = \underline{\left(x - \frac{1}{2}\right)^2}$

$(a-b)^2$ où $a = x$, $b = \frac{1}{2}$

r) $\underbrace{\frac{xy}{3} + \frac{y^2}{9} + \frac{x^2}{4}}_{(a+b)^2} = \underline{\left(\frac{x}{2} + \frac{y}{3}\right)^2}$

$(a+b)^2$ où $a = \frac{x}{2}$, $b = \frac{y}{3}$

s) $(a+b)^2 - 2(a+b)c + c^2 = \underline{(a+b-c)^2}$

u) $x^2(a+b) + 2(a+b)x + a+b = (a+b)(x^2 + 2x + 1) = \underline{(a+b)(x+1)^2}$

2.2.3 Factoriser :

a) $x^{12} - 125$

b) $a^4 - \dfrac{8ab^3}{27}$

c) $27c^3 + \dfrac{1}{64}$

d) $z^3 + 8a^3b^6$

e) $z^6 + 27$

f) $z^3 - 6z^2 + 12z - 8$

g) $1 + 9a + 27a^2 + 27a^3$

h) $x^3 + x^2y + \dfrac{xy^2}{3} + \dfrac{y^3}{27}$

i) $12a^3 + \dfrac{9ab^2}{4} + \dfrac{3b^3}{16} + 9a^2b$

Rappel : $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

a) $x^{12} - 125 = (x^4)^3 - (5)^3 = \underline{(x^4 - 5)(x^8 + 5x^4 + 25)}$

b) $a^4 - \dfrac{8ab^3}{27} = a\left(a^3 - \dfrac{8b^3}{27}\right) = a\left(a^3 - \left(\dfrac{2b}{3}\right)^3\right)$

$= \underline{a\left(a - \dfrac{2b}{3}\right)\left(a^2 + \dfrac{2ab}{3} + \dfrac{4b^2}{9}\right)}$

c) $27c^3 + \dfrac{1}{64} = (3c)^3 + \left(\dfrac{1}{4}\right)^3 = \underline{\left(3c + \dfrac{1}{4}\right)\left(9c^2 - \dfrac{3c}{4} + \dfrac{1}{16}\right)}$

d) $z^3 + 8a^3b^6 = z^3 + (2ab^2)^3 = \underline{(z + 2ab^2)(z^2 - 2zab^2 + 4a^2b^4)}$

e) $z^6 + 27 = (z^2)^3 + (3)^3 = \underline{(z^2 + 3)(z^4 - 3z^2 + 9)}$

f) $\underbrace{z^3 - 6z^2 + 12z - 8}_{(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3} = \underline{(z-2)^3}$

g) $\underbrace{1 + 9a + 27a^2 + 27a^3}_{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3} = \underline{(1+3a)^3}$

h) $\underbrace{x^3 + x^2y + \frac{xy^2}{3} + \frac{y^3}{27}}_{(a+b)^3 \text{ où } a = x,\ b = \frac{y}{3}} = \underline{\left(x + \frac{y}{3}\right)^3}$

i) $12a^3 + \frac{9ab^2}{4} + \frac{3b^3}{16} + 9a^2b = 12\left(a^3 + \frac{9ab^2}{12\cdot 4} + \frac{3b^3}{12\cdot 16} + \frac{9a^2b}{12}\right)$

$= 12\underbrace{\left(a^3 + \frac{3ab^2}{16} + \frac{b^3}{64} + \frac{3a^2b}{4}\right)}_{(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \text{ où } x = a,\ y = \frac{b}{4}} = \underline{12\left(a + \frac{b}{4}\right)^3}$

2.2.4 Factoriser :

a) x^2+5x+6 e) $9x^2+6x+1$ i) $6x^2+5x+1$ m) $40x^2+3x-28$

b) x^2+5x+4 f) $4z^2+5z+1$ j) $x^2-22x+85$ n) $a^2+9a-10$

c) u^2-6u+8 g) $x^2-2x-80$ k) x^2+x+1 o) $2x^2-5x-2$

d) $x^2-2x-35$ h) $3y^2+7y+3$ l) $16u^2-72u+81$ p) $4m^2+25m-21$

Rappel :

Méthode : Trinôme unitaire du second degré :

$$\boxed{x^2+bx+c=(x+\alpha)(x+\beta)}=x^2+\alpha\cdot x+\beta\cdot x+\alpha\cdot\beta=x^2+(\alpha+\beta)x+\alpha\cdot\beta$$

$\Rightarrow$ Il faut que : $\boxed{\alpha\cdot\beta=c}$ et que $\boxed{\alpha+\beta=b}$

On trouve α et β par tâtonnement.

a) x^2+5x+6 $\rightarrow$ on cherche α et β tq : $\begin{cases}\alpha+\beta=5\\ \alpha\cdot\beta=6\end{cases}\Rightarrow\begin{cases}\alpha=3\\ \beta=2\end{cases}$

$\Rightarrow x^2+5x+6=\underline{(x+3)(x+2)}$

b) x^2+5x+4 $\rightarrow$ $\begin{cases}\alpha+\beta=5\\ \alpha\cdot\beta=4\end{cases}\Rightarrow\begin{cases}\alpha=1\\ \beta=4\end{cases}$

$\Rightarrow x^2+5x+4=\underline{(x+1)(x+4)}$

c) u^2-6u+8 $\rightarrow$ $\begin{cases}\alpha+\beta=-6\\ \alpha\cdot\beta=8\end{cases}\Rightarrow\begin{cases}\alpha=-2\\ \beta=-4\end{cases}$

$\Rightarrow u^2-6u+8=\underline{(u-2)(u-4)}$

d) $x^2 - 2x - 35 \rightarrow \begin{cases} \alpha + \beta = -2 \\ \alpha \cdot \beta = -35 \end{cases} \Rightarrow \begin{cases} \alpha = -7 \\ \beta = +5 \end{cases}$

$\Rightarrow x^2 - 2x - 35 = \underline{(x-7)(x+5)}$

e) $9x^2 + 6x + 1 = (3x+1)^2$

$(a+b)^2$

f) $4z^2 + 5z + 1$

Pour factoriser, on résout l'équation : $4z^2 + 5z + 1 = 0$

$\Delta = 25 - 4 \cdot 4 \cdot 1 = 9 = 3^2$

$\Rightarrow z_1 = \frac{-5-3}{8} = -1$

$z_2 = \frac{-5+3}{8} = -\frac{1}{4}$

$\Rightarrow S = \left\{-1 ; -\frac{1}{4}\right\}$

$\Rightarrow 4z^2 + 5z + 1 = 4\left(z+1\right)\left(z+\frac{1}{4}\right) = 4(z+1)\left(\frac{4z+1}{4}\right) = \underline{(z+1)(4z+1)}$

Rappel : $P(x) = ax^2 + bx + c$, si $\Delta > 0 \rightarrow$ 2 zéros

$\rightarrow P(x) = a(x - x_1)(x - x_2)$

g) $x^2 - 2x - 80 \rightarrow$ on cherche $\begin{cases} \alpha + \beta = -2 \\ \alpha \cdot \beta = -80 \end{cases} \Rightarrow \begin{cases} \alpha = -10 \\ \beta = 8 \end{cases}$

$\Rightarrow x^2 - 2x - 80 = \underline{(x-10)(x+8)}$

h) $3y^2 + 7y + 3 \rightarrow$ on résout l'équation : $3y^2 + 7y + 3 = 0$

$\Delta = 49 - 4 \cdot 3 \cdot 3 = 49 - 36 = 13$

$\rightarrow y_1 = \frac{-7-\sqrt{13}}{6}$, $y_2 = \frac{-7+\sqrt{13}}{6}$

$\rightarrow \; 3y^2+7y+3 = 3\left(y+\frac{7+\sqrt{13}}{6}\right)\left(y+\frac{7-\sqrt{13}}{6}\right)$

i) $6x^2+5x+1$ → on résout l'équation : $6x^2+5x+1=0$

$\Delta = 25-4\cdot6\cdot1 = 1$

$\Rightarrow x_1 = \frac{-5-1}{12} = -\frac{1}{2}$, $x_2 = \frac{-5+1}{12} = \frac{-4}{12} = -\frac{1}{3}$

$\Rightarrow 6x^2+5x+1 = 6\left(x+\frac{1}{2}\right)\left(x+\frac{1}{3}\right) = \cancel{6}\left(\frac{2x+1}{\cancel{2}}\right)\left(\frac{3x+1}{\cancel{3}}\right)$

$= \underline{(2x+1)(3x+1)}$

j) $x^2-22x+85$ → on cherche $\begin{cases} \alpha+\beta = -22 \\ \alpha\cdot\beta = 85 \end{cases} \Rightarrow \begin{cases} \alpha = -17 \\ \beta = -5 \end{cases}$

$\Rightarrow x^2-22x+85 = \underline{(x-17)(x-5)}$

k) x^2+x+1 → On résout $x^2+x+1=0$

$\Delta = 1-4\cdot1\cdot1 < 0$ → impossible

$\Rightarrow x^2+x+1 = \underline{x^2+x+1}$

l) $16u^2-72u+81 = \underbrace{(4u)^2 - 2\cdot9\cdot4u + (9)^2}_{a^2-2ab+b^2} = \underbrace{\underline{(4u-9)^2}}_{(a-b)^2}$

m) $40x^2+3x-28$ → on résout l'équation : $40x^2+3x-28=0$

$\Delta = 9-4\cdot40\cdot(-28) = 4489 = 67^2$

$\rightarrow x_1 = \frac{-3-67}{80} = -\frac{7}{8}$, $x_2 = \frac{-3+67}{80} = \frac{64}{80} = \frac{8}{10} = \frac{4}{5}$

$\Rightarrow 40x^2+3x-28 = 40\left(x+\frac{7}{8}\right)\left(x-\frac{4}{5}\right) = \cancel{40}\left(\frac{8x+7}{\cancel{8}}\right)\left(\frac{5x-4}{\cancel{5}}\right) = \underline{(8x+7)(5x-4)}$

2.2.5 Factoriser :

a) $x^4 - 13x^2 + 36$

b) $a^6 + 19a^3 - 216$

c) $x^8 - 257x^4 + 256$

d) $7x^4 - 61x^2 - 18$

e) $64x^6 - 91x^3 + 27$

f) $6x^4 + 7x^2 - 3$

g) $16x^8 - 641x^4 + 625$

h) $81z^4 + 80z^2 - 1$

a) $x^4 - 13x^2 + 36 \rightarrow$ on pose $y = x^2$

$\rightarrow y^2 - 13y + 36 \rightarrow$ on cherche $\begin{cases} \alpha + \beta = -13 \\ \alpha \cdot \beta = 36 \end{cases} \Rightarrow \begin{cases} \alpha = -9 \\ \beta = -4 \end{cases}$

$\rightarrow x^4 - 13x^2 + 36 = y^2 - 13y + 36 = (y-9)(y-4) = (x^2-9)(x^2-4)$

$= \underline{(x-3)(x+3)(x-2)(x+2)}$

b) $a^6 + 19a^3 - 216 \Rightarrow$ on pose $x = a^3$

$\Rightarrow a^6 + 19a^3 - 216 = x^2 + 19x - 216 \rightarrow$ on cherche $\begin{cases} \alpha + \beta = 19 \\ \alpha \cdot \beta = -216 \end{cases}$

$\Rightarrow \begin{cases} \alpha = 27 \\ \beta = -8 \end{cases}$

$\Rightarrow a^6 + 19a^3 - 216 = x^2 + 19x - 216 = (x+27)(x-8) = (a^3+27)(a^3-8)$

$= \underline{(a+3)(a^2-3a+9)(a-2)(a^2+2a+4)}$

c) $x^8 - 257x^4 + 256 \rightarrow$ on pose $x^4 = y$

$\rightarrow x^8 - 257x^4 + 256 = y^2 - 257y + 256 \rightarrow \begin{cases} \alpha + \beta = -257 \\ \alpha \cdot \beta = 256 \end{cases}$

$\Rightarrow \begin{cases} \alpha = -256 \\ \beta = -1 \end{cases}$

$$\Rightarrow x^8 - 257x^4 + 256 = y^2 - 257y + 256 = (y-256)(y-1)$$

$$= (x^4 - 256)(x^4 - 1) = (x^4 - 4^4)(x^2-1)(x^2+1)$$

$$= (x^2 - 4^2)(x^2 + 4^2)(x-1)(x+1)(x^2+1)$$

$$= (x-4)(x+4)(x^2+16)(x-1)(x+1)(x^2+1)$$

$$= \underline{(x^2+16)(x+4)(x-4)(x^2+1)(x-1)(x+1)}$$

d) $7x^4 - 61x^2 - 18$ → on pose $x^2 = y$

→ $7x^4 - 61x^2 - 18 = 7y^2 - 61y - 18$ → on résout l'équation :

$$7y^2 - 61y - 18 = 0$$

$$\Delta = 61^2 - 4 \cdot 7(-18) = 3721 + 504 = 4225 = 65^2$$

→ $y_1 = \dfrac{61-65}{14} = -\dfrac{4}{14} = -\dfrac{2}{7}$

$y_2 = \dfrac{61+65}{14} = \dfrac{126}{14} = 9$

$$\Rightarrow 7x^4 - 61x^2 - 18 = 7y^2 - 61y - 18 = 7\left(y + \frac{2}{7}\right)(y-9)$$

$$= 7\left(x^2 + \frac{2}{7}\right)(x^2 - 9) = \cancel{7}\left(\frac{7x^2+2}{\cancel{7}}\right)(x-3)(x+3)$$

$$\Rightarrow 7x^4 - 61x^2 - 18 = \underline{(7x^2+2)(x-3)(x+3)}$$

e) $64x^6 - 91x^3 + 27$ → on pose $x^3 = y$

→ $64x^6 - 91x^3 + 27 = 64y^2 - 91y + 27$ → on résout l'équation :

$$64y^2 - 91y + 27 = 0$$

$$\Delta = 91^2 - 4 \cdot 64 \cdot 27 = 8281 - 6912 = 1369 = 37^2$$

$$\Rightarrow y_1 = \frac{91 - 37}{128} = \frac{54}{128} = \frac{27}{64}$$

$$y_2 = \frac{91 + 37}{128} = \frac{128}{128} = 1$$

$$\Rightarrow 64x^6 - 91x^3 + 27 = 64y^2 - 91y + 27 = 64\left(y - \frac{27}{64}\right)\left(y - 1\right)$$

$$= 64\left(x^3 - \frac{27}{64}\right)\left(x^3 - 1\right) = \cancel{64}\left(\frac{64x^3 - 27}{\cancel{64}}\right)\left(x^3 - 1\right)$$

$$= (64x^3 - 27)(x-1)(x^2 + x + 1)$$

$$= \left((4x)^3 - 3^3\right)(x-1)(x^2 + x + 1)$$

$$= (4x - 3)(16x^2 + 12x + 9)(x-1)(x^2 + x + 1)$$

g) $6x^4 + 7x^2 - 3$ → on pose $y = x^2$

→ $6x^4 + 7x^2 - 3 = 6y^2 + 7y - 3$ → on résout l'équation :

$$6y^2 + 7y - 3 = 0$$

$$\Delta = 49 - 4 \cdot 6 \cdot (-3) = 49 + 72 = 121 = 11^2$$

$$\Rightarrow y_1 = \frac{-7 - 11}{12} = -\frac{18}{12} = -\frac{3}{2}$$

$$y_2 = \frac{-7 + 11}{12} = \frac{4}{12} = \frac{1}{3}$$

$$\Rightarrow 6x^4 + 7x^2 - 3 = 6y^2 + 7y - 3 = 6\left(y + \frac{3}{2}\right)\left(y - \frac{1}{3}\right) = 6\left(x^2 + \frac{3}{2}\right)\left(x^2 - \frac{1}{3}\right)$$

$$= \cancel{6}\left(\frac{2x^2 + 3}{\cancel{2}}\right)\left(\frac{3x^2 - 1}{\cancel{3}}\right) = (2x^2 + 3)(3x^2 - 1)$$

$$= (2x^2 + 3)(\sqrt{3}x - 1)(\sqrt{3}x + 1)$$

2.2.6 Factoriser : Méthode des groupements !

a) $ax + bx + ay + by$

b) $a + b + ax + bx + ay + by$

c) $ax - bx - ay + by$

d) $ax - 4x + 4y - ay$

e) $ax + x - a - 1$

f) $x^3 + x - x^2 - 1$

g) $\frac{xy}{2} - \frac{x}{4} + \frac{yz}{3} - \frac{z}{6}$

h) $10xz - 10z - x^2 + x$

i) $a^2 - 2ab + b^2 - 1$

j) $4x^2 + 2x - 9y^2 - 3y$

k) $1 + x + x^2 + x^3 + x^4 + x^5$

l) $8y^4 - 8y^3 + y - 1$

m) $x^3 + x - x^2 - 1$

n) $2a^4 - 3 - 2a^3 + 3a$

a) $ax + bx + ay + by = x(a+b) + y(a+b) = \underline{(a+b)(x+y)}$

b) $a + b + ax + bx + ay + by = (a+b) + x(a+b) + y(a+b)$
$= \underline{(a+b)(1 + x + y)}$

c) $ax - bx - ay + by = x(a-b) - y(a-b) = \underline{(a-b)(x-y)}$

d) $ax - 4x + 4y - ay = x(a-4) - y(a-4) = \underline{(a-4)(x-y)}$

e) $ax + x - a - 1 = x(a+1) - (a+1) = \underline{(a+1)(x-1)}$

f) $x^3 + x - x^2 - 1 = x(x^2+1) - (x^2+1) = \underline{(x^2+1)(x-1)}$

g) $\frac{xy}{2} - \frac{x}{4} + \frac{yz}{3} - \frac{z}{6} = \frac{x}{2}\left(y - \frac{1}{2}\right) + \frac{z}{3}\left(y - \frac{1}{2}\right)$
$= \underline{\left(y - \frac{1}{2}\right)\left(\frac{x}{2} + \frac{z}{3}\right)}$

h) $10xz - 10z - x^2 + x = 10z(x-1) - x(x-1) = \underline{(x-1)(10z - x)}$

i) $a^2 - 2ab + b^2 - 1 = (a-b)^2 - 1 = \underline{(a-b-1)(a-b+1)}$

j) $4x^2 + 2x - 9y^2 - 3y = 4x^2 - 9y^2 + 2x - 3y = (2x-3y)(2x+3y) + 2x - 3y$

$= \underline{(2x-3y)(2x+3y+1)}$

k) $1 + x + x^2 + x^3 + x^4 + x^5 = (1 + x + x^2) + (x^3 + x^4 + x^5)$

$= (1 + x + x^2) + x^3(1 + x + x^2) = (1 + x + x^2)(1 + x^3)$

$= \underline{(1 + x + x^2)(1 + x)(1 - x + x^2)}$

l) $8y^4 - 8y^3 + y - 1 = 8y^3(y-1) + (y-1) = (y-1)(8y^3 + 1)$

$= (y-1)((2y)^3 + 1) = \underline{(y-1)(2y+1)(4y^2 - 2y + 1)}$

m) $x^3 + x - x^2 - 1 = x^3 - x^2 + x - 1 = x^2(x-1) + x - 1$

$= \underline{(x-1)(x^2+1)}$

n) $2a^4 - 3 - 2a^3 + 3a = 2a^4 - 2a^3 + 3a - 3 = 2a^3(a-1) + 3(a-1)$

$= \underline{(a-1)(2a^3 + 3)}$

o) $6x^2 + xy + 18xz + 3yz = x(6x + y) + 3z(6x + y)$

$= \underline{(6x + y)(x + 3z)}$

2.2.7 Décomposer en facteurs après avoir groupé.

a) $x - 2y - x^2 + 2xy + (x-2y)^2$

b) $2x^2 + 3x - 10xy - 15y$

c) $3x^3 - 20y^2z - 5z + 12x^3y^2$

d) $8x + (2x+3y)(x-2y) - 6x^2 + 12y - 9xy + (2x+3y)^2$

e) $\dfrac{xz}{2} - \dfrac{x}{4} + \dfrac{yz}{3} - \dfrac{y}{6}$

f) $x^5 - \frac{4}{5}x^2y - \frac{5}{4}x^3z + yz$

g) $\frac{2}{9}x^2y^3 - \frac{1}{20}x^2 + \frac{40}{27}y^3 - \frac{1}{3}$

h) $3x^4y^3z + x^4y^3 + 3x^3y^4z - 3x^2y^5z - x^2y^5$

i) $x^{3m+2} - 2x^{m+2}y^m + x^{2m}y^{m+3} - 2y^{2m+3}$ avec $m \in \mathbb{N}^\star$

j) $x^{3m+1} - x^{2m+1}y^{2n} + 2x^my^{3n} - 2y^{5n}$ avec $m, n \in \mathbb{N}^\star$

a) $x - 2y - x^2 + 2xy + (x-2y)^2 = (x-2y) - x(x-2y) + (x-2y)^2$

$= (x-2y)(1 - x + x - 2y) = \underline{(x-2y)(1-2y)}$

b) $2x^2 + 3x - 10xy - 15y = 2x^2 - 10xy + 3x - 15y$

$= 2x(x-5y) + 3(x-5y) = \underline{(x-5y)(2x+3)}$

c) $3x^3 - 20y^2z - 5z + 12x^3y^2 = 3x^3 + 12x^3y^2 - 20y^2z - 5z$

$= 3x^3(1+4y^2) - 5z(1+4y^2) = \underline{(1+4y^2)(3x^3-5z)}$

d) $8x + (2x+3y)(x-2y) - 6x^2 + 12y - 9xy + (2x+3y)^2$

$= 8x - 6x^2 + 12y - 9xy + (2x+3y)(x-2y) + (2x+3y)^2$

$= 2x(4-3x) + 3y(4-3x) + (2x+3y)(x-2y) + (2x+3y)^2$

$= (4-3x)(2x+3y) + (2x+3y)(x-2y) + (2x+3y)^2$

$$= (2x+3y)\left(4-\cancel{3x}+\cancel{x}-2y+\cancel{2x}+3y\right)$$

$$= (2x+3y)(y+4)$$