1MS + 1MR 2024/2025

Algèbre - Corrigé

2.1 Développer une expression

2.1.1 Effectuer et réduire :

a) $3 + (xz + y^2)$ $= 3 + xz + y^2$

b) $3 - (xz + y^2)$ $= 3 - xz + y^2$

c) $3(xz + y^2)$ $= 3xz + 3y^2$

d) $(2a + b - c) + (3a - b + c)$
$= 2a + b - c + 3a - b + c = 5a$

e) $(2a + b - c) - (3a - b + c)$
$= 2a + b - c - 3a + b - c = -a + 2b - 2c$

f) $(2a + b - c)(3a - b + c)$

g) $(x^3 - 2x^2 - 5) + (-4x^3 - 1)$

h) $(x^3 - 2x^2 - 5) - (-4x^3 - 1)$

i) $(x^3 - 2x^2 - 5)(-4x^3 - 1)$

j) $\left(u + \frac{v}{4}\right) + \left(\frac{3u}{4} - \frac{5v}{6}\right)$

k) $\left(u + \frac{v}{4}\right) - \left(\frac{3u}{4} - \frac{5v}{6}\right)$

l) $\left(u + \frac{v}{4}\right)\left(\frac{3u}{4} - \frac{5v}{6}\right)$

f) $(2a + b - c)(3a - b + c) = 6a^2 - 2ab + 2ac + 3ab - b^2 + bc - 3ac + bc - c^2$

$= \underline{6a^2 - b^2 - c^2 + ab - ac + 2bc}$

g) $(x^3 - 2x^2 - 5) + (-4x^3 - 1) = x^3 - 2x^2 - 5 - 4x^3 - 1 = \underline{-3x^3 - 2x^2 - 6}$

h) $(x^3 - 2x^2 - 5) - (-4x^3 - 1) = x^3 - 2x^2 - 5 + 4x^3 + 1 = \underline{5x^3 - 2x^2 - 4}$

i) $(x^3 - 2x^2 - 5)(-4x^3 - 1) = -4x^6 - x^3 + 8x^5 + 2x^2 + 20x^3 + 5$

$= \underline{-4x^6 + 8x^5 + 19x^3 + 2x^2 + 5}$

j) $\left(u + \frac{v}{4}\right) + \left(\frac{3u}{4} - \frac{5v}{6}\right) = \frac{12u + 3v + 9u - 10v}{12}$

$= \underline{\frac{21u - 7v}{12}}$

k) $\left(u + \frac{v}{4}\right) - \left(\frac{3u}{4} - \frac{5v}{6}\right) = \frac{12u + 3v - 9u + 10v}{12} = \underline{\frac{3u + 13v}{12}}$

e) $\left(u+\frac{v}{4}\right)\left(\frac{3u}{4}-\frac{5v}{6}\right)=\frac{3u^2}{4}-\frac{5uv}{6}+\frac{3uv}{16}-\frac{5v^2}{24}$

$$=\frac{12\cdot 3u^2-8\cdot 5uv+9uv-10v^2}{48}=\frac{36u^2-40uv+9uv-10v^2}{48}$$

$$=\frac{36u^2-31uv-10v^2}{48}$$

2.1.2 Effectuer et réduire :

a) $(a+b)^2$

b) $(a-b)^2$

c) $(a+b)(a-b)$

d) $(a+b)^3$

e) $(a-b)^3$

f) $(a-b)(a^2+ab+b^2)$

g) $(a+b)(a^2-ab+b^2)$

a) $(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = \underline{a^2 + 2ab + b^2}$

b) $(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = \underline{a^2 - 2ab + b^2}$

c) $(a+b)(a-b) = a^2 - ab + ba - b^2 = \underline{a^2 - b^2}$

d) $(a+b)^3 = (a+b)^2(a+b) = (a^2 + 2ab + b^2)(a+b)$

$= a^3 + a^2b + 2a^2b + 2ab^2 + b^2a + b^3$

$= \underline{a^3 + 3a^2b + 3ab^2 + b^3}$

e) $(a-b)^3 = (a-b)^2(a-b) = (a^2 - 2ab + b^2)(a-b)$

$= a^3 - a^2b - 2a^2b + 2ab^2 + b^2a - b^3$

$= \underline{a^3 - 3a^2b + 3ab^2 - b^3}$

f) $(a-b)(a^2 + ab + b^2) = a^3 + \cancel{a^2b} + \cancel{ab^2} - \cancel{ba^2} - \cancel{ab^2} - b^3$

$= \underline{a^3 - b^3}$

g) $(a+b)(a^2 - ab + b^2) = a^3 - \cancel{a^2b} + \cancel{ab^2} + \cancel{ba^2} - \cancel{ab^2} + b^3$

$= \underline{a^3 + b^3}$

2.1.3 Effectuer et réduire :

a) $(a+8)^2$

b) $(y^4-3b)^3$

c) $(u-3)(u+3)$

d) $(2m-5n)(4m^2+10mn+25n^2)$

e) $(7-f)^2$

f) $(4+2z^2)^3$

g) $(3+y^3)(y^6-3y^3+9)$

h) $(x^2+y^2)(x^2-y^2)$

i) $(t+3u^5)^3$

j) $(2x-7)^2$

k) $(b^2-c^3)(b^2c^3+b^4+c^6)$

l) $(a-3b)^3$

a) $(a+8)^2 = (a+8)(a+8) = a^2+8a+8a+64 = \underline{a^2+16a+64}$

b) $(y^4-3b)^3 = (y^4-3b)(y^4-3b)(y^4-3b) = (y^8-3y^4b-3y^4b+9b^2)(y^4-3b)$

$= (y^8-6y^4b+9b^2)(y^4-3b) = y^{12}-3y^8b-6y^8b+18y^4b^2+9y^4b^2-27b^3$

$= \underline{y^{12}-9y^8b+27y^4b^2-27b^3}$

c) $(u-3)(u+3) = u^2+3u-3u-9 = \underline{u^2-9}$

d) $(2m-5n)(4m^2+10mn+25n^2)$

$= 8m^3+20m^2n+50mn^2-20m^2n-50mn^2-125n^3$

$= \underline{8m^3-125n^3}$

i) $(t+3u^5)^3 = (t+3u^5)(t+3u^5)(t+3u^5)$

$= (t^2+3tu^5+3tu^5+9u^{10})(t+3u^5) = (t^2+6tu^5+9u^{10})(t+3u^5)$

$= t^3+3t^2u^5+6t^2u^5+18tu^{10}+9tu^{10}+27u^{15}$

$= \underline{t^3+9t^2u^5+27tu^{10}+27u^{15}}$

f) $(4+2z^2)^3 = (4+2z^2)(4+2z^2)(4+2z^2) = (16+8z^2+8z^2+4z^4)(4+2z^2)$

$= (16+16z^2+4z^4)(4+2z^2) = 64+32z^2+64z^2+32z^4+16z^4+8z^6$

$= \underline{8z^6+48z^4+96z^2+64}$

g) $(3+y^3)(y^6-3y^3+9) = \cancel{3y^6}-\cancel{9y^3}+27+y^9-\cancel{3y^6}+\cancel{9y^3}$

$= \underline{y^9+27}$

h) $(x^2+y^2)(x^2-y^2) = x^4-x^2y^2+y^2x^2-y^4 = \underline{x^4-y^4}$

k) $(b^2-c^3)(b^2c^3+b^4+c^6) = \cancel{b^4c^3}+b^6+\cancel{b^2c^6}-\cancel{b^2c^6}-\cancel{b^4c^3}-c^9$

$= \underline{b^6-c^9}$

l) $(a-3b)^3 = (a-3b)(a-3b)(a-3b) = (a^2-3ab-3ab+9b^2)(a-3b)$

$= (a^2-6ab+9b^2)(a-3b) = a^3-3a^2b-6a^2b+18ab^2+9ab^2-27b^3$

$= \underline{a^3-9a^2b+27ab^2-27b^3}$

2.1.4 Réduire au maximum.

a) $(x-1)^2-(y+1)^2$

b) $(1+x)^2-(1-x)^2$

c) $\left(\frac{1}{2}\,x+\frac{1}{2}\,y\right)^2-\left(\frac{1}{2}\,x-\frac{1}{2}\,y\right)^2$

$a-b$ ✓

d) $(2\,x+y)^2+(2\,x-y)^2-2\,(2\,x+y)(2\,x-y)$

e) $(3\,x+y)(3\,x-y)-(3\,x+2\,y)^2-(x-3\,y)^2$

f) $(x+2)^2-(x+1)^2-(x+1)(x-1)-x\,(x+4)-4$

g) $(x+y)(x-y)+(x-y)^2-(x+y)^2+y\,(4\,x+y)$

h) $(x^2+4\,y^2)(x+2\,y)(x-2\,y)-(x^2-2\,y^2)^2$

i) $(3\,x-2\,y)^2+(4\,x+y)(4\,x-y)-(5\,x-3\,y)^2+6\,y(y-3\,x)$

j) $(2\,x-y)^2\,(2\,x+y)^2-(x-2\,y)^2(x+2\,y)^2-15\,(x+y)(x-y)(x^2+y^2)$

a) $(x-1)^2-(y+1)^2=\left[(x-1)-(y+1)\right]\left[(x-1)+(y+1)\right]=(x-y-2)(x+y)$

$= x^2+\cancel{xy}-\cancel{yx}-y^2-2x-2y = \underline{x^2-y^2-2x-2y}$

b) $(1+x)^2-(1-x)^2=\left[(1+x)-(1-x)\right]\left[(1+x)+(1-x)\right]=(2x)(2)=\underline{4x}$

c) $\left(\frac{1}{2}x+\frac{1}{2}y\right)^2-\left(\frac{1}{2}x-\frac{1}{2}y\right)^2=\left[\left(\frac{1}{2}x+\frac{1}{2}y\right)-\left(\frac{1}{2}x-\frac{1}{2}y\right)\right]\left[\left(\frac{1}{2}x+\frac{1}{2}y\right)+\left(\frac{1}{2}x-\frac{1}{2}y\right)\right]$

$=\left(\cancel{\frac{1}{2}x}+\frac{1}{2}y-\cancel{\frac{1}{2}x}+\frac{1}{2}y\right)\left(\frac{1}{2}x+\cancel{\frac{1}{2}y}+\frac{1}{2}x-\cancel{\frac{1}{2}y}\right)=\underline{xy}$

d) $(2x+y)^2+(2x-y)^2-2(2x+y)(2x-y)=\left[(2x+y)-(2x-y)\right]^2$

$=\left(\cancel{2x}+y-\cancel{2x}+y\right)^2=(2y)^2=\underline{4y^2}$

e) $(3x+y)(3x-y)-(3x+2y)^2-(x-3y)^2$

$=9x^2-y^2-(9x^2+12xy+4y^2)-(x^2-6xy+9y^2)$

$= 9x^2 - y^2 - 9x^2 - 12xy - 4y^2 - x^2 + 6xy - 9y^2$

$= \underline{-x^2 - 6xy - 14y^2}$

f) $(x+2)^2 - (x+1)^2 - (x+1)(x-1) - x(x+4) - 4$

$= [(x+2)-(x+1)][(x+2)+(x+1)] - (x^2-1) - x^2 - 4x - 4$

$= (\cancel{x}+2-\cancel{x}-1)(x+2+x+1) - x^2 + 1 - x^2 - 4x - 4$

$= 1\cdot(2x+3) - 2x^2 - 3 - 4x$

$= 2x + \cancel{3} - 2x^2 - \cancel{3} - 4x = \underline{-2x^2 - 2x}$

g) $(x+y)(x-y) + (x-y)^2 - (x+y)^2 + y(4x+y)$

$= x^2 - \cancel{y^2} + x^2 - 2xy + \cancel{y^2} - (x^2 + 2xy + y^2) + 4xy + y^2$

$= 2x^2 + 2xy + y^2 - x^2 - 2xy - y^2 = \underline{x^2}$

h) $(x^2+4y^2)(x+2y)(x-2y) - (x^2-2y^2)^2$

$= (x^2+4y^2)(x^2-4y^2) - (x^4 - 4x^2y^2 + 4y^4)$

$= \cancel{x^4} - 16y^4 - \cancel{x^4} + 4x^2y^2 - 4y^4 = \underline{4x^2y^2 - 20y^4}$

i) $(3x-2y)^2 + \underbrace{(4x+y)(4x-y)}_{(a-b)(a+b) = a^2-b^2} - (5x-3y)^2 + 6y(y-3x)$

$= 9x^2 - 12xy + 4y^2 + 16x^2 - y^2 - (25x^2 - 30xy + 9y^2) + 6y^2 - 18xy$

$= 9x^2 + 16x^2 + 4y^2 - y^2 - 12xy - 25x^2 + 30xy - 9y^2 + 6y^2 - 18xy$

$= \underbrace{9x^2 + 16x^2 - 25x^2}_{} + \underbrace{4y^2 - y^2 - 9y^2 + 6y^2}_{} \underbrace{- 12xy + 30xy - 18xy}_{}$

$= 0 + 0 + 0 = 0$

j) $(2x-y)^2(2x+y)^2-(x-2y)^2(x+2y)^2-15(x+y)(x-y)(x^2+y^2)$

$= [\underbrace{(2x-y)(2x+y)}_{(a-b)(a+b)}]^2 - [\underbrace{(x-2y)(x+2y)}_{(a-b)(a+b)}]^2 - 15(\underbrace{x^2-y^2}_{(a-b)(a+b)})(x^2+y^2)$

$= (4x^2-y^2)^2 - (x^2-4y^2)^2 - 15(x^4-y^4)$

$= 16x^4 - 8x^2y^2 + y^4 - (x^4 - 8x^2y^2 + 16y^4) - 15x^4 + 15y^4$

$= 16x^4 - \cancel{8x^2y^2} + y^4 - x^4 + \cancel{8x^2y^2} - 16y^4 - 15x^4 + 15y^4$

$= \underbrace{16x^4 - x^4 - 15x^4}_{0} + \underbrace{y^4 + 15y^4 - 16y^4}_{0} = 0$

2.1.5 Réduire au maximum.

a) $-(6\,ab^2 - 7\,x^3)(6\,ab^2 + 7\,x^3)$

b) $(4\,x^2 - 7\,y^3)^2 - (x^2 - 5\,y^2)(4\,x^2 + y^3)$

c) $(3\,x - 2\,y)^2 - (4\,x + 5\,y)^2 - 2\,(2\,x - y)(3\,x - 5\,y)$

d) $(2\,a - 3\,b)^3 - (2\,a - 3\,b)^2 - (2\,a - 3\,b)$

a) $-\underbrace{(6ab^2-7x^3)(6ab^2+7x^3)}_{(a-b)(a+b)} = -(36a^2b^4-49x^6) = \underline{-36a^2b^4+49x^6}$

$$\left(\begin{array}{l} 6ab^2\cdot 6ab^2 = (6ab^2)^2 = 36a^2b^4 \\ (7x^3)^2 = 49x^6 \end{array}\right)$$

b) $(4x^2-7y^3)^2-(x^2-5y^2)(4x^2+y^3)$

$= 16x^4-56x^2y^3+49y^6-(4x^4+x^2y^3-20x^2y^2-5y^5)$

$= 16x^4-56x^2y^3+49y^6-4x^4-x^2y^3+20x^2y^2+5y^5$

$= \underline{12x^4-57x^2y^3+20x^2y^2+5y^5+49y^6}$

c) $(3x-2y)^2-(4x+5y)^2-2(2x-y)(3x-5y)$

$= (9x^2-12xy+4y^2)-(16x^2+40xy+25y^2)-2(6x^2-10xy-3xy+5y^2)$

$= 9x^2-12xy+4y^2-16x^2-40xy-25y^2-12x^2+20xy+6xy-10y^2$

$= \underline{-19x^2-26xy-31y^2}$

d) $(2a-3b)^3-(2a-3b)^2-(2a-3b)$

$= (2a-3b)\left((2a-3b)^2-(2a-3b)-1\right)$

$= (2a-3b)(4a^2-12ab+9b^2-2a+3b-1)$

$= 8a^3-24a^2b+18ab^2-4a^2+6ab-2a-12a^2b+36ab^2-27b^3+6ab-9b^2+3b$

$= \underline{8a^3-36a^2b+54ab^2-4a^2-27b^3+12ab-9b^2-2a+3b}$

2.1.6 Soit $p(x) = 2\,x^3 - 3\,x^2 + 5\,x - 1$ et $q(x) = 3\,x^3 + 2\,x^2 - 4\,x + 2$. Déterminer

a) le polynôme $p + q$

b) le degré du polynôme $p \cdot q$, ainsi que le coefficient de son terme de degré 4.

a) $p(x) + q(x) = 2x^3 - 3x^2 + 5x - 1 + 3x^3 + 2x^2 - 4x + 2$

$\Rightarrow p(x) + q(x) = \underline{5x^3 - x^2 + x + 1}$

b) $p(x) \cdot q(x) = 2x^3 \cdot 3x^3 + \cdots + 1x^4 + \cdots$

$= 6x^6 + \cdots + 1 \cdot x^4 + \cdots$

degré 6 du $p \cdot q$

coefficient de son terme de degré 4

2.1.7 Soit $p(x) = x^2 + x + 2$ et $q(x) = x^3 - 2\,x$. Déterminer les polynômes

$$p + q, \quad p - q, \quad \text{et} \quad p \cdot q$$

* $p + q = x^2 + x + 2 + x^3 - 2x = \underline{x^3 + x^2 - x + 2}$
* $p - q = x^2 + x + 2 - x^3 + 2x = \underline{-x^3 + x^2 + 3x + 2}$
* $p \cdot q = (x^2 + x + 2)(x^3 - 2x) = x^5 - 2x^3 + x^4 - 2x^2 + 2x^3 - 4x$

$= \underline{x^5 + x^4 - 2x^2 - 4x}$

2.1.8 Soit les polynômes

$a(x) = 3\,x^2 - 4\,x + 3$, $p(x) = x^4 + 2\,x^3 - 2\,x^2 - 4\,x + 17$ et $q(x) = 2\,x^3 - 3\,x^2 - 5\,x + 18$

a) calculer et réduire au maximum $(a(x))^2$
b) calculer $p - q$
c) déterminer le degré du polynôme $p \cdot q$
d) déterminer le coefficient du polynôme $p \cdot q$ de degré 7
e) déterminer le coefficient du polynôme $p \cdot q$ de degré 4

a) $(a(x))^2 = (3x^2 - 4x + 3)^2 = (3x^2 - 4x + 3)(3x^2 - 4x + 3)$

$= 9x^4 - 12x^3 + 9x^2 - 12x^3 + 16x^2 - 12x + 9x^2 - 12x + 9$

$= \underline{9x^4 - 24x^3 + 34x^2 - 24x + 9}$

b) $p - q = x^4 + 2x^3 - 2x^2 - 4x + 17 - (2x^3 - 3x^2 - 5x + 18)$

$= x^4 + \cancel{2x^3} - 2x^2 - 4x + 17 - \cancel{2x^3} + 3x^2 + 5x - 18$

$= \underline{x^4 + x^2 + x - 1}$

c) degré du $p \cdot q$: $x^4 \cdot x^3 = x^7$ $\rightarrow$ $\underline{\text{degré } 7}$

d) coefficient du polynôme $p \cdot q$ de degré 7 : $x^4 \cdot 2x^3 = 2x^7$ $\rightarrow$ $\underline{2}$

e) $p \cdot q$: coefficient $p \cdot q$ degré 4 : $(x^4 + 2x^3 - 2x^2 - 4x + 17)(2x^3 - 3x^2 - 5x + 18)$

$= 18x^4 - 10x^4 + 6x^4 - 8x^4 = 6x^4$ $\rightarrow$ $\underline{6}$

2.1.9 Effectuer et réduire.

a) $(2\,x-y-z)-(3\,x+2\,y-3\,z)-(4\,x+y-z)+(5\,x+4\,y-4\,z)$

b) $\left(\frac{1}{2}\,x^3-\frac{1}{6}\,x^2y\right)-\left(x^3-\frac{1}{8}xy^2-\frac{1}{10}y^3\right)+\left(\frac{1}{2}\,x^3-\frac{1}{2}\,x^2y+\frac{1}{8}\,xy^2+\frac{1}{10}\,y^3\right)$
$-\left(-\frac{2}{3}\,x^2y-\frac{3}{4}xy^2-\frac{4}{5}y^3\right)$

c) $x^2y-\{-[2xy^2+7x^3+5x^2y]+4xy^2\}-6x^2y$

d) $2\,xy+3\,y^2-\left\{-\frac{1}{2}\,x^2y+\left[\frac{1}{2}\,y^2-\left(3\,xy+\frac{5}{4}\,x^2y\right)\right]-\left(\frac{1}{8}\,x^2y-4\,y^2\right)\right\}$

e) $\left(\frac{1}{3}\,x^2\right)^3-x^4-\left\{\frac{3}{4}\,x^2y^2-\left(\frac{1}{2}\,x^3\right)^2+\left[(-2\,x)^4+\frac{1}{4}\,x^2y^2-\frac{8}{27}\,x^6\right]\right\}$

f) $(3\,x^2-x+2)(4\,x+3)(2\,x-1)$

g) $(x-3)(x+4)(x-5)(x+6)$

h) $x(x+1)-3\,x(-x+3)+2(x^2-x)$

i) $[x(x+y)-y(x-y)]\,(x+y)-xy(x+y)$

j) $(x+y)(x-2\,y)(2\,x-y)-(2\,x+y)(x-2\,y)(x-y)$

a) $(2x-y-z)-(3x+2y-3z)-(4x+y-z)+(5x+4y-4z)$

$= 2x-y-z-3x-2y+3z-4x-y+z+5x+4y-4z$

$= \underline{-z}$

f) $(3x^2-x+2)(4x+3)(2x-1) = (3x^2-x+2)(8x^2-4x+6x-3)$

$= (3x^2-x+2)(8x^2+2x-3) = 24x^4+6x^3-9x^2-8x^3-2x^2+3x+16x^2+4x-6$

$= \underline{24x^4-2x^3+5x^2+7x-6}$

i) $[x(x+y)-y(x-y)](x+y)-xy(x+y)$

$= [x^2+\cancel{xy}-\cancel{yx}+y^2](x+y)-xy(x+y)$

$= (x+y)[x^2+y^2-xy] = \underline{x^3+y^3}$

j) $(x+y)(x-2y)(2x-y)-(2x+y)(x-2y)(x-y)$

$= (x-2y)[(x+y)(2x-y)-(2x+y)(x-y)]$

$= (x-2y)[2x^2-xy+2xy-y^2-(2x^2-2xy+yx-y^2)]$

$$= (x-2y)\left[\cancel{2x^2} - \cancel{y^2} + xy - \cancel{2x^2} + xy + \cancel{y^2}\right]$$

$$= (x-2y)\,2xy = \underline{2x^2y - 4xy^2}$$

2.1.10 Réduire.

a) $(x-1)^3 - (x-1)(x+1)(x-3)$
b) $(x+1)(x-1)^2 - (x-2)^3$
c) $(x^2+2\,x+1)^2 - 4\,x(x^2+1) - (x^2+1)(x^2-1)$
d) $(x+y)^3 - (x-y)^3 - (x^3-y^3) - (x-y)(x^2+xy+y^2)$
e) $19(x^3+y^3) - (3\,x-2\,y)^3 - (3\,y-2\,x)^3 - 18\,xy(x+y)$
f) $x^4+y^4+(x^2+y^2+2\,xy)^2 - 2(x^2+y^2+xy)^2$
g) $(2\,x-3\,y)^3 - 3\,y(x-3\,y)^2 - 9\,xy(4\,y-x)$
h) $[(x-y)(x+y)+(2\,x-y)^2]^3$
i) $(x^3+x^2+x+1)(x^3-x^2+x-1)$
j) $[(x-y)(x-y)]^2 - (x^2+y^2)^2 + 4\,xy\,[(x-y)^2+xy+1]$

a) $(x-1)^3 - (x-1)(x+1)(x-3) = (x-1)\left((x-1)^2 - (x+1)(x-3)\right)$

$$= (x-1)\left(x^2-2x+1-\left(x^2-3x+x-3\right)\right) = (x-1)\left(\cancel{x^2}-\cancel{2x}+1-\cancel{x^2}+\cancel{2x}+3\right)$$

$$= (x-1)\,4 = \underline{4x-4}$$

b) $(x+1)(x-1)^2 - (x-2)^3 = (x^2-1)(x-1) - (x-2)^3$

$$= x^3 - x^2 - x + 1 - (x^3 - 6x^2 + 12x - 8)$$

$$= \cancel{x^3} - x^2 - x + 1 - \cancel{x^3} + 6x^2 - 12x + 8 = \underline{5x^2 - 13x + 9}$$

c) $(x^2+2x+1)^2 - 4x(x^2+1) - (x^2+1)(x^2-1)$

$$= (x^2+2x+1)(x^2+2x+1) - 4x(x^2+1) - (x^4-1)$$

$$= \cancel{x^4} + \cancel{2x^3} + x^2 + \cancel{2x^3} + 4x^2 + 2x + x^2 + 2x + 1 - \cancel{4x^3} - 4x - \cancel{x^4} + 1 = \underline{6x^2+2}$$