

Exercice 2.4

$$(u \cdot v)' = u'v + u \cdot v'$$

Calculer les dérivées des fonctions ci-dessous en utilisant la règle du produit (réponse sous forme factorisée).

$$a) f(x) = \underbrace{(2x+1)}_u \cdot \underbrace{(3x-2)}_v$$

$$f'(x) = 2(3x-2) + (2x+1) \cdot 3 = 6x-4+6x+3$$

$$f'(x) = 12x - 1$$

$$b) f(x) = x \cdot (x^2+1)^2$$

$$u = x \rightarrow u' = 1 \quad v = (x^2+1)^2 \rightarrow v' = 2(x^2+1)^{2-1} \cdot (x^2)' = 2(x^2+1) \cdot 2x$$

$$f'(x) = 1(x^2+1)^2 + x \cdot 2(x^2+1) \cdot 2x$$

$$f'(x) = (x^2+1)^2 + 4x^2(x^2+1)$$

$$f'(x) = (x^2+1) [(x^2+1) + 4x^2]$$

$$f'(x) = (x^2+1)(5x^2+1)$$

$$c) f(x) = (x+3) \cdot (x-1)^3$$

$$u = x+3 \rightarrow u' = 1 \quad v = (x-1)^3 \rightarrow v' = 3(x-1)^{3-1} \cdot 1 = 3(x-1)^2$$

$$f'(x) = 1 \cdot (x-1)^3 + (x+3) \cdot 3(x-1)^2$$

$$f'(x) = (x-1)^2 [(x-1) + (x+3) \cdot 3]$$

$$f'(x) = (x-1)^2 (x-1 + 3x+9)$$

$$f'(x) = (x-1)^2 (4x+8)$$

$$f'(x) = 4(x-1)^2 (x+2)$$

$$d) f(x) = x^4 \cdot (2-3x)^2$$

$$u = x^4 \rightarrow u' = 4x^3$$

$$v = (2-3x)^2 \rightarrow v' = 2(2-3x)^{2-1} \cdot (0-3) = -6(2-3x)$$

$$f'(x) = 4x^3(2-3x)^2 + x^4 \cdot [-6(2-3x)]$$

$$f'(x) = 4x^3(2-3x)^2 - 6x^4(2-3x)$$

$$f'(x) = 2x^3(2-3x)[2(2-3x) - 3x]$$

$$f'(x) = 2x^3(2-3x)(4-6x-3x)$$

$$f'(x) = 2x^3(2-3x)(4-9x)$$

$$e) f(x) = (x-1)^2 \cdot (2x+1)^3$$

$$u = (x-1)^2 \rightarrow u' = 2 \cdot (x-1)^{2-1} \cdot 1 = 2(x-1)$$

$$v = (2x+1)^3 \rightarrow v' = 3(2x+1)^{3-1} \cdot (2+0) = 3(2x+1)^2 \cdot 2 = 6(2x+1)^2$$

$$f'(x) = 2(x-1)(2x+1)^3 + (x-1)^2 \cdot 6(2x+1)^2$$

$$f'(x) = 2(x-1)(2x+1)^2 [(2x+1) + 3(x-1)]$$

$$f'(x) = 2(x-1)(2x+1)^2(2x+1+3x-3)$$

$$f'(x) = 2(x-1)(2x+1)^2(5x-2)$$

$$f) f(x) = (x-3)^3 \cdot (x^2-4)^2$$

$$u = (x-3)^3 \rightarrow u' = 3(x-3)^{3-1} \cdot (1-0) = 3(x-3)^2$$

$$v = (x^2-4)^2 \rightarrow v' = 2(x^2-4)^{2-1} \cdot (2x-0) = 4x(x^2-4)$$

$$f'(x) = 3(x-3)^2(x^2-4)^2 + (x-3)^3 \cdot 4x(x^2-4)$$

$$f'(x) = (x-3)^2(x^2-4)[3(x^2-4) + 4x(x-3)]$$

$$f'(x) = (x-3)^2(x^2-4)(3x^2-12+4x^2-12x)$$

$$f'(x) = (x-3)^2(x^2-4)(7x^2-12x-12)$$

$$(x^2-4) = (x+2)(x-2) \Rightarrow f'(x) = (x-3)^2(x+2)(x-2)(7x^2-12x-12)$$