CORRIGÉ

Chapitre 2

Exponentielles et logarithmes

Equations

2.1 Résoudre les équations ci-dessous :

a) $5^x = 25$

b) $3^x = \frac{1}{9}$

c) $x^4 = 16$

d) $4^x = 64$

e) $4^x = 8$

f) $9^{2x+1} = 1$

g) $16 \cdot 2^x = 4^{3x+5}$

h) $8^{7x-2} = 8^{-3x+8}$

i) $\left(\frac{1}{2}\right)^{x+7} = 2$

a) $5^x = 25$

$\Leftrightarrow 5^x = 5^2 \Leftrightarrow x = 2 \Rightarrow \boxed{S = \{2\}}$

b) $3^x = \frac{1}{9}$

$\Leftrightarrow 3^x = \frac{1}{3^2} \Leftrightarrow 3^x = 3^{-2} \Leftrightarrow x = -2 \Rightarrow \boxed{S = \{-2\}}$

c) $x^4 = 16$

$\Leftrightarrow x^4 - 16 = 0 \Leftrightarrow \underbrace{(x^2)^2 - (2^2)^2}_{a^2 - b^2 = (a-b)(a+b)} = 0 \Leftrightarrow (x^2-4)(x^2+4) = 0$

$\Leftrightarrow \underset{x=2}{(x-2)}\underset{x=-2}{(x+2)}(\underbrace{x^2+4}_{>0}) = 0 \Rightarrow x = \pm 2 \Rightarrow \boxed{S = \{-2 ; 2\}}$

d) $4^x = 64$

$\Leftrightarrow 2^{2x} = 2^6 \Leftrightarrow 2x = 6 \Leftrightarrow x = 3 \Rightarrow \boxed{S = \{3\}}$

e) $4^x = 8$

$\Leftrightarrow 2^{2x} = 2^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2} \Rightarrow \boxed{S = \left\{\frac{3}{2}\right\}}$

f) $9^{2x+1} = 1$

$\Leftrightarrow 9^{2x+1} = 9^0 \Leftrightarrow 2x+1 = 0 \Leftrightarrow x = -\frac{1}{2} \Rightarrow \boxed{S = \left\{-\frac{1}{2}\right\}}$

g) $16 \cdot 2^x = 4^{3x+5}$

$\Leftrightarrow 2^4 \cdot 2^x = 2^{2(3x+5)} \Leftrightarrow 2^{4+x} = 2^{6x+10}$

$\Leftrightarrow 4 + x = 6x + 10 \Leftrightarrow 6x - x = 4 - 10 \Leftrightarrow 5x = -6$

$\Rightarrow x = -\frac{6}{5} \Rightarrow \boxed{S = \left\{-\frac{6}{5}\right\}}$

h) $8^{7x-2} = 8^{-3x+8}$

$\Leftrightarrow 7x - 2 = -3x + 8 \Leftrightarrow 7x + 3x = 8 + 2 \Leftrightarrow 10x = 10$

$\Rightarrow x = 1 \Rightarrow \boxed{S = \{1\}}$

i) $\left(\frac{1}{2}\right)^{x+7} = 2$

$\Leftrightarrow \left(2^{-1}\right)^{x+7} = 2^1 \Leftrightarrow 2^{-(x+7)} = 2^1$

$\Leftrightarrow -(x+7) = 1 \Leftrightarrow -x - 7 = 1 \Rightarrow -x = 8 \Rightarrow x = -8$

$\Rightarrow \boxed{S = \{-8\}}$

2.3 Résoudre les équations ci-dessous :

a) $145^x = 3451$ b) $5^{2x} = 456.35$ c) $1000 \cdot 1.12^x = 10'000$

d) $20 \cdot 5^{3x} = 800$ e) $\dfrac{e^{x+1}}{100} = 20$ f) $20 + 100 \cdot e^{-0.5x} = 60$

Rappel : • $a^x = u \iff \log_a(u) = x$

• $\log_b(u) = \dfrac{\log_a(u)}{\log_a(b)} = \dfrac{\log(u)}{\log(b)} = \dfrac{\ln(u)}{\ln(b)}$

a) $145^x = 3451$

$\iff \log_{145}(3451) = x \Rightarrow x = \dfrac{\log(3451)}{\log(145)} \cong 1,6369$

$\Rightarrow \boxed{S \cong \{1,6369\}}$

b) $5^{2x} = 456,35$

$\iff 2x = \log_5(456,35) = \dfrac{\log(456,35)}{\log(5)}$

$\Rightarrow x = \dfrac{1}{2} \dfrac{\log(456,35)}{\log(5)} \cong 1,902297694$

$\Rightarrow \boxed{S \cong \{1,9023\}}$

c) $1'000 \cdot 1{,}12^x = 10'000$

$\Leftrightarrow \quad 1{,}12^x = \frac{10'000}{1000} = 10 \quad \Leftrightarrow \quad x = \log_{1{,}12}(10)$

$x = \frac{\log(10)}{\log(1{,}12)} \cong 20{,}31776056 \Rightarrow \boxed{S \cong \{20{,}31776\}}$

d) $20 \cdot 5^{3x} = 800$

$\Leftrightarrow \quad 5^{3x} = \frac{800}{20} = 40$

$\Leftrightarrow \quad 3x = \log_5(40) \quad \Leftrightarrow \quad x = \frac{1}{3}\log_5(40)$

$\Leftrightarrow \quad x = \frac{1}{3}\,\frac{\log(40)}{\log(5)} \cong 0{,}76401 \Rightarrow \boxed{S \cong \{0{,}76401\}}$

e) $\frac{e^{x+1}}{100} = 20$

$\Leftrightarrow \quad e^{x+1} = 20 \cdot 100 = 2'000$

$\Leftrightarrow \quad \ln(2000) = x+1 \quad \Leftrightarrow \quad x = \ln(2000) - 1$

$\Rightarrow x \cong 6{,}60090 \Rightarrow \boxed{S \cong \{6{,}60090\}}$

f) $20 + 100 \cdot e^{-0,5x} = 60$

$$\Leftrightarrow \quad 100 \cdot e^{-0,5x} = 60 - 20 = 40$$

$$\Leftrightarrow \quad e^{-0,5x} = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$$

$$\Leftrightarrow \quad e^{-0,5x} = \frac{2}{5} \quad \Leftrightarrow \quad \underbrace{\ln(e)^{-0,5x}}_{= -0,5x} = \ln\left(\frac{2}{5}\right)$$

$$\Rightarrow \quad -0,5x = \ln\left(\frac{2}{5}\right) \quad \Rightarrow \quad -\frac{1}{2}x = \ln\left(\frac{2}{5}\right)$$

$$\Rightarrow \quad x = -2\ln\left(\frac{2}{5}\right) \simeq 1,832581$$

$$\Rightarrow \quad \boxed{S \simeq \{1,832581\}}$$