4.2.1 Résoudre les équations ci-dessous :

a) $7^{x+6} = 7^{3x+4}$

b) $6^{7-x} = 6^{2x+1}$

c) $3^{2x+3} = 3^{(x^2)}$

d) $9^{(x^2)} = 3^{3x+2}$

e) $2^{-100x} = 0,5^{x-4}$

f) $(\frac{1}{4})^{6-x} = 4$

g) $27^{x-1} = 9^{2x-3}$

h) $2^x \cdot 4^x = -5$

i) $(5^{x-2})^4 = 125 \cdot 5^{5x-3}$

j) $(3^{x-1})^3 = 9 \cdot 3^{x-2}$

k) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

l) $5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

a) $7^{x+6} = 7^{3x+4} \Leftrightarrow x+6 = 3x+4 \Rightarrow x = 1 \Rightarrow S = \{1\}$

b) $6^{7-x} = 6^{2x+1} \Leftrightarrow 7-x = 2x+1 \Rightarrow x = 2 \Rightarrow S = \{2\}$

c) $3^{2x+3} = 3^{(x^2)} \Leftrightarrow 2x+3 = x^2 \Leftrightarrow x^2 - 2x - 3 = 0$

$\Leftrightarrow (x-3)(x+1) = 0 \Rightarrow S = \{-1; 3\}$

d) $9^{(x^2)} = 3^{3x+2} \Leftrightarrow (3^2)^{(x^2)} = 3^{3x+2} \Leftrightarrow 2x^2 = 3x+2$

$\Leftrightarrow 2x^2 - 3x - 2 = 0 \Rightarrow (2x+1)(x-2) = 0$

$\Rightarrow S = \{-\frac{1}{2}; 2\}$

e) $2^{-100x} = 0,5^{x-4} \Leftrightarrow 2^{-100x} = (\frac{1}{2})^{x-4} \Leftrightarrow 2^{-100x} = (2^{-1})^{(x-4)}$

$\Leftrightarrow 2^{-100x} = 2^{-(x-4)} \Leftrightarrow -100x = -(x-4)$

$\Leftrightarrow 100x = 2 - 4 \Rightarrow x = -\frac{4}{99}$

$\Rightarrow S = \{-\frac{4}{99}\}$

1) $\left(\frac{1}{4}\right)^{6-x} = 4 \iff \left(4^{-1}\right)^{(6-x)} = 4$

$\iff 4^{-(6-x)} = 4^{1} \iff -(6-x) = 1$

$\iff -6+x = 1 \Rightarrow x = 7 \Rightarrow \boxed{S = \{7\}}$

g) $27^{x-1} = 9^{2x-3} \iff \left(3^{3}\right)^{(x-1)} = \left(3^{2}\right)^{(2x-3)}$

$\iff 3^{3(x-1)} = 3^{2(2x-3)} \iff 3(x-1) = 2(2x-3)$

$\iff 3x-3 = 4x-6 \Rightarrow x = 3 \Rightarrow \boxed{S = \{3\}}$

h) $2^{x} \cdot 4^{x} = -5 \iff 2^{x} \cdot 2^{2x} = -5$

$\iff \underbrace{2^{3x}}_{\text{für exp} > 0} = -5 \Rightarrow \boxed{S = \emptyset}$

i) $\left(5^{x-2}\right)^{4} = 125 \cdot 5^{5x-3}$

$\Rightarrow 5^{(x-2)\cdot 4} = 5^{3} \cdot 5^{5x-3} \iff 5^{4(x-2)} = 5^{3+5x-3}$

$\Rightarrow 4(x-2) = 5x \iff 4x-8 = 5x \Rightarrow -x = 8$

$\Rightarrow x = -8 \Rightarrow \boxed{S = \{-8\}}$

j) $\left(3^{x-1}\right)^{3} = 9 \cdot 3^{x-2} \Rightarrow 3^{(x-1)\cdot 3} = 3^{2} \cdot 3^{x-2}$

$\Rightarrow 3^{3(x-1)} = 3^{2+x-2} \iff 3(x-1) = x$

$\Rightarrow 3x-3 = x \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

$\Rightarrow \boxed{S = \left\{\frac{3}{2}\right\}}$

k) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

$\Leftrightarrow 3^{2(2x+1)} - 36 \cdot 3^{2x+1} = -243$ (*)

$\Leftrightarrow$ poser $3^{2x+1} = y$

(*) $\Leftrightarrow y^2 - 36y + 243 = 0 \Leftrightarrow (y-9)(y-27) = 0$

$\Leftrightarrow y = 9$ ou $y = 27$

* $y = 9 = 3^2 \Rightarrow 3^{2x+1} = 3^2 \Leftrightarrow 2x+1 = 2 \Leftrightarrow x_1 = \frac{1}{2}$

* $y = 27 = 3^3 \Leftrightarrow 3^{2x+1} = 3^3 \Leftrightarrow 2x+1 = 3 \Leftrightarrow x_2 = 1$

$\Leftrightarrow \boxed{S = \left\{ \frac{1}{2} ; 1 \right\}}$

l) $5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

$\Leftrightarrow 5^{4x-7+1} - 120 \cdot 5^{2x-3} = 625$

$\Leftrightarrow 5^{4x-6} - 120 \cdot 5^{2x-3} = 625$

$\Leftrightarrow 5^{2(2x-3)} - 120 \cdot 5^{2x-3} = 625$ (*)

poser $y = 5^{2x-3}$

(*) $\Leftrightarrow y^2 - 120y - 625 = 0 \Leftrightarrow (y-125)(y+5) = 0$

* $y = 125 = 5^3 \Rightarrow 5^{2x-3} = 5^3 \Leftrightarrow x_1 = 3$

* $y = -5 = \underbrace{5^{2x-3}}_{>0}$ $\Leftrightarrow$ impossible

donc $\boxed{S = \{3\}}$

4.2.4 Simplifier les expressions ci-dessous sans utiliser la machine :

a) $\log(16) + 2\log(3) - 2\log(2) - \frac{1}{2}\log(9)$ b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

c) $4\log(5) + \log\left(\frac{1}{5}\right) - 3\log(3) + \frac{1}{3}\log(27)$ d) $\dfrac{\log(20) + \log(100) - \log(2)}{\log(5'000) - \log(5) + \log(0,1)}$

a) $\log(16) + 2\log(3) - 2\log(2) - \frac{1}{2}\log(9)$

$= \log(2^4) + 2\log(3) - 2\log(2) - \frac{1}{2}\log(3^2)$

$= 4\log(2) + 2\log(3) - 2\log(2) - \log(3)$

$= \boxed{2\log(2) + \log(3)}$

b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

$= \log(3\cdot 5) + 3\log(2\cdot 5) - \log(6\cdot 5) - \log(5)$

$= \log(3) + \log(5) + 3\log(2) + 3\log(5) - \log(6) - \log(5) - \log(5)$

$\log(6) = \log(2\cdot 3)$

$= \log(3) + 3\log(2) + 2\log(5) - \log(2) - \log(3)$

$= 2\log(2) + 2\log(5) = 2\left(\log(2) + \log(5)\right) = 2\log(2\cdot 5)$

$= 2\log(10) = \boxed{2}$

c) $4\log(5) + \log\left(\frac{1}{5}\right) - 3\log(3) + \frac{1}{3}\log(27)$

$= 4\log(5) - \log(5) - 3\log(3) = \boxed{3\log(5) - 2\log(3)}$

d) $\dfrac{\log(20) + \log(100) - \log(2)}{\log(5000) - \log(5) + \log(0{,}1)}$

$= \dfrac{2\log(2) + \log(5) + 2 - \log(2)}{3 + \log(5) - \log(5) - 1} = \dfrac{\log(2) + \log(5) + 2}{2}$

$= \dfrac{\log(10) + 2}{2} = \boxed{\dfrac{3}{2}}$

4.2.5 Résoudre les équations ci-dessous :

a) $x = \log_2(32)$ b) $2^x = 100$ c) $\log_x(256) = 4$ d) $\log_2(x) = 4$

e) $10^x = 5$ f) $e^{2x-1} = 27$ g) $\log_x(1'000) = 3$ h) $12^x = -49$

a) $x = \log_2(32) \Leftrightarrow 2^x = 32 \Leftrightarrow 2^x = 2^5 \Leftrightarrow x = 5$

$\Rightarrow \boxed{S = \{5\}}$

b) $2^x = 100 \Rightarrow x = \log_2(100) \Rightarrow \boxed{S = \{\log_2(100)\}}$

c) $\log_x(256) = 4 \Leftrightarrow x^4 = 256 \Leftrightarrow x = \sqrt[4]{256} = 4$

$\Rightarrow \boxed{S = \{4\}}$

d) $\log_2(x) = 4 \Leftrightarrow 2^4 = x \Rightarrow x = 16 \Rightarrow \boxed{S = \{16\}}$

e) $10^x = 5 \Leftrightarrow \log(10^x) = \log(5)$

$\Rightarrow x = \log(5) \Rightarrow \boxed{S = \{\log(5)\}}$

f) $e^{2x-1} = 27 \Leftrightarrow \ln(e^{2x-1}) = \ln(27) \Leftrightarrow 2x - 1 = \ln(27)$

$\Rightarrow x = \dfrac{\ln(27) + 1}{2} \Rightarrow \boxed{S = \left\{\dfrac{\ln(27) + 1}{2}\right\}}$

g) $\log_x(1000) = 3 \iff x^3 = 1000 = 10^3$

$\Rightarrow x = 3 \Rightarrow \boxed{S = \{3\}}$

h) $\underbrace{12^x}_{>0} = -49 \Rightarrow \boxed{S = \emptyset}$

4.2.6 Résoudre les équations ci-dessous :

a) $\log_{11}(x+1) = \log_{11}(7)$ b) $\log_6(2x-3) = \log_6(12) - \log_6(3)$

c) $\log(x) - \log(x+1) = 3\log(4)$ d) $2\log_3(x) = 3\log_3(5)$

e) $\ln(x) + \ln(x-2) = 0,5\ln(9)$ f) $\log_8(x+4) = 1 - \log_8(x-3)$

a) $\log_{11}(x+1) = \log_{11}(7)$ (*)

Condition : $x+1 > 0 \iff x > -1$

$\Rightarrow$ ED = $]-1 ; +\infty[$

(*) $\iff x+1 = 7 \iff x = 6 \Rightarrow$ $\boxed{S = \{6\}}$

b) $\log_6(2x-3) = \log_6(12) - \log_6(3)$ (*)

condition : $2x-3 > 0 \iff x > \frac{3}{2} \Rightarrow$ ED = $]\frac{3}{2} ; +\infty[$

(*) $\iff \log_6(2x-3) = \log_6\left(\frac{12}{3}\right) = \log_6(4)$

$\iff 2x-3 = 4 \iff 2x = 7 \Rightarrow x = \frac{7}{2}$

$\Rightarrow$ $\boxed{S = \left\{\frac{7}{2}\right\}}$

c) $\log(x) - \log(x+1) = 3\log(4)$

$\iff \log\left(\frac{x}{x+1}\right) = \log(4^3)$ (*)

$\Rightarrow$ ED = $\mathbb{R}_+^*$

(*) $\iff \frac{x}{x+1} = 4^3 = 64 \iff x = 64(x+1)$

$\Rightarrow x = -\frac{64}{63} \notin$ ED

$\Rightarrow$ $\boxed{S = \emptyset}$

d) $2\log_3(x) = 3\log_3(5)$ (*)

$\Rightarrow$ ED : $\mathbb{R}_+^*$

(*) $\Leftrightarrow$ $\log_3(x^2) = \log_3(5^3) \Leftrightarrow x^2 = 5^3 \Leftrightarrow x^2 = 125$

$\Rightarrow x = \sqrt{125} = 5\sqrt{5}$

$\Rightarrow$ $\boxed{S = \{5\sqrt{5}\}}$

e) $\ln(x) + \ln(x-2) = 0{,}5\ln(9)$ (*)

conditions : $\begin{cases} x > 0 \\ x-2 > 0 \end{cases}$ $\Rightarrow$ ED : $]2;+\infty[$

(*) $\Leftrightarrow$ $\ln(x(x-2)) = \ln(9)^{1/2} = \ln(\sqrt{9})$

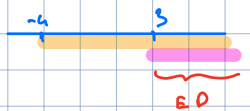
$\Leftrightarrow$ $x(x-2) = 3 \Leftrightarrow x^2 - 2x - 3 = 0$

$\Leftrightarrow$ $(x-3)(x+1) = 0$ $x = -1$ à éliminer

Donc $\boxed{S = \{3\}}$

f) $\log_8(x+4) = 1 - \log_8(x-3)$ (*)

conditions : $\begin{cases} x+4 > 0 \\ x-3 > 0 \end{cases} \Leftrightarrow \begin{cases} x > -4 \\ x > 3 \end{cases}$



$\Rightarrow$ ED : $]3;+\infty[$

(*) $\Leftrightarrow$ $\log_8(x+4) + \log_8(x-3) = 1$

$\Leftrightarrow$ $\log_8[(x+4)(x-3)] = \log_8(8) \Leftrightarrow (x+4)(x-3) = 8$

$\Leftrightarrow$ $x^2 - 3x + 4x - 12 - 8 = 0 \Leftrightarrow x^2 + x - 20 = 0$

$\Leftrightarrow$ $(x+5)(x-4) = 0$ $\Rightarrow x = -5$ à éliminer !

Donc $\boxed{S = \{4\}}$

4.2.7 Résoudre les systèmes d'équations :

a) $\begin{cases} \log(x)+\log(y) &= 2 \\ x+y &= 25 \end{cases}$ b) $\begin{cases} \log(x)-\log(y) &= 1 \\ xy &= 2 \end{cases}$

a) $ED = \mathbb{R}_+^* \times \mathbb{R}_+^*$

$$\Leftrightarrow \begin{cases} \log(x)+\log(y) = 2 \\ x+y = 25 \end{cases} \Leftrightarrow \begin{cases} \log(xy) = 2 \\ x+y = 25 \end{cases}$$

$$\Leftrightarrow \begin{cases} \log(xy) = \log(10^2) \\ x+y = 25 \end{cases} \Leftrightarrow \begin{cases} xy = 100 \\ x+y = 25 \end{cases}$$

$$\Rightarrow x(25-x) = 100 \Leftrightarrow x^2 - 25x + 100 = 0$$

$$\Leftrightarrow (x-20)(x-5) = 0$$

$\Rightarrow x_1 = 20 \Rightarrow y_1 = 5$

$x_2 = 5 \Rightarrow y_2 = 20$

$\Rightarrow \boxed{S = \{(5;20);(20;5)\}}$

b) $ED = \mathbb{R}_+^* \times \mathbb{R}_+^*$

$$\Rightarrow \begin{cases} \log(x)-\log(y) = 1 \\ xy = 2 \end{cases} \Leftrightarrow \begin{cases} \log\left(\frac{x}{y}\right) = \log(10) \\ xy = 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{x}{y} = 10 \\ xy = 2 \end{cases} \Rightarrow \begin{cases} x = 10y \\ xy = 2 \end{cases} \Rightarrow 10y^2 = 2$$

$$\Rightarrow y^2 = \frac{1}{5} \Rightarrow y = \frac{\sqrt{5}}{5} \quad (y>0)$$

$$\Rightarrow x = 10 \cdot \frac{\sqrt{5}}{5} = 2\sqrt{5}$$

$$\Rightarrow \boxed{S = \left\{\left(2\sqrt{5}; \frac{\sqrt{5}}{5}\right)\right\}}$$