

## Généralités sur les fonctions :

### Corrigé

2.2.9 Les fonctions  $f$  suivantes sont des fonctions composées. Donner une décomposition possible de  $f$  en deux fonctions :  $f = g \circ h$ .

a)  $f(x) = \sqrt{3x+1}$

b)  $f(x) = \frac{1}{x^2+x+3}$

c)  $f(x) = (x+2)^7$

d)  $f(x) = \frac{\sqrt{x}+2}{\sqrt{x}-4}$

e)  $f(x) = \log(x^2+4)$

f)  $f(x) = 3^{2x-5}$

Rappel:  $(g \circ f)(x) = g(f(x))$

a)  $f(x) = \sqrt{3x+1}$

$$f(x) = (g \circ h)(x) = g(h(x))$$

$$\Rightarrow \underline{g(x) = \sqrt{x} \quad \text{et} \quad h(x) = 3x+1}$$

En effet :  $g(h(x)) = g(3x+1) = \sqrt{3x+1} = f(x)$

b)  $f(x) = \frac{1}{x^2+x+3}$

$$f(x) = (g \circ h)(x) = g(h(x)) = g(x^2+x+3) = \frac{1}{x^2+x+3}$$

$$\text{d'où} \quad \underline{g(x) = \frac{1}{x} \quad \text{et} \quad h(x) = x^2+x+3}$$

c)  $f(x) = (x+2)^7$

$$\Rightarrow g(x) = x^7 \quad \text{et} \quad h(x) = x+2$$

d)  $f(x) = \frac{\sqrt{x}+2}{\sqrt{x}-4}$   $\Rightarrow \underline{g(x) = \frac{x+2}{x-4} \quad \text{et} \quad h(x) = \sqrt{x}}$

$$e) f(x) = \log(x^2 + 4)$$

$$\Rightarrow \underline{g(x) = \log(x) \quad \text{et} \quad h(x) = x^2 + 4}$$

$$f) f(x) = 3^{2x-5}$$

$$\Rightarrow \underline{g(x) = 3^x \quad \text{et} \quad h(x) = 2x - 5}$$

2.2.10 Tracer le graphe des fonctions suivantes :

a)  $f(x) = 2$

b)  $f(x) = \frac{2}{5}x$

c)  $f(x) = x + 4$

d)  $f(x) = 3x - 6$

e)  $f(x) = -2x + 3$

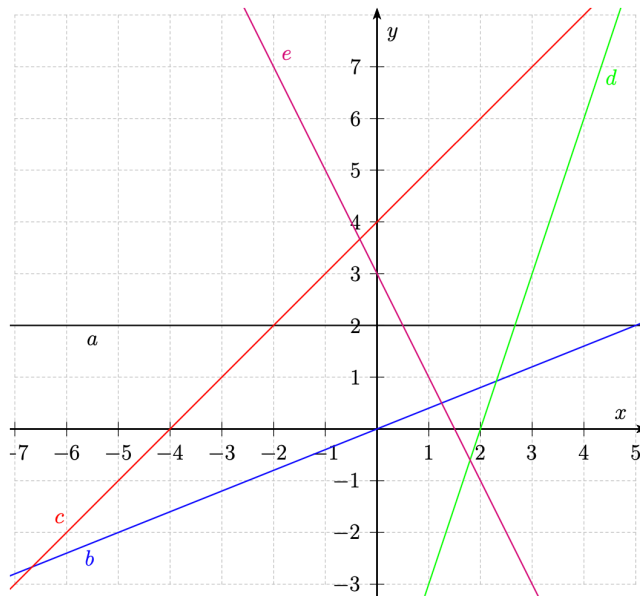
f)  $f(x) = x^2 + x - 2$

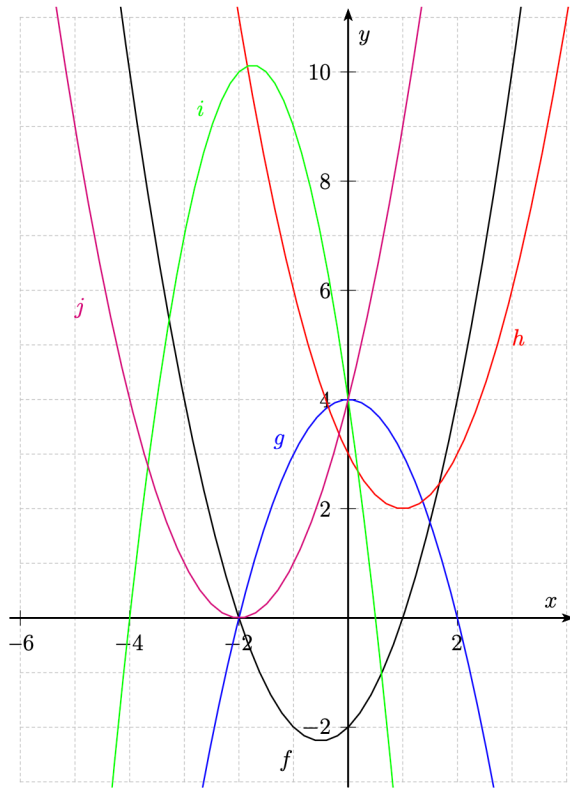
g)  $f(x) = 4 - x^2$

h)  $f(x) = x^2 - 2x + 3$

i)  $f(x) = -2x^2 - 7x + 4$

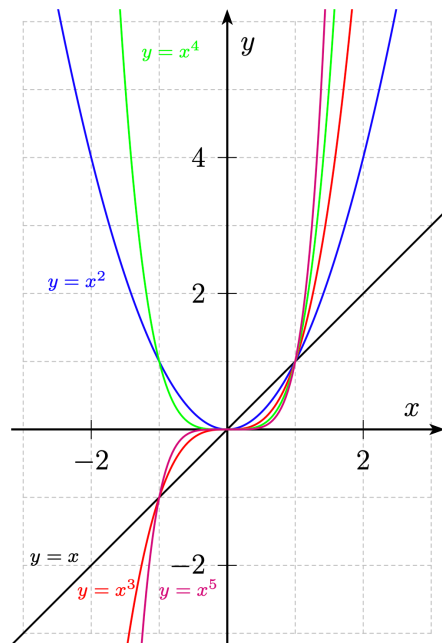
j)  $f(x) = x^2 + 4x + 4$





**2.2.11** Tracer dans le même système d'axes les graphes des fonctions suivantes :

$$f_1(x) = x, f_2(x) = x^2, f_3(x) = x^3, f_4(x) = x^4, f_5(x) = x^5$$



2.2.12 Esquisser le graphe des fonctions données par :

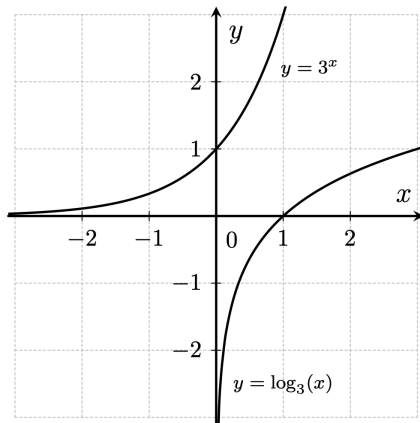
a)  $f(x) = 3^x$  et  $g(x) = \log_3(x)$

b)  $f(x) = \left(\frac{1}{2}\right)^x$  et  $g(x) = \log_{\frac{1}{2}}(x)$

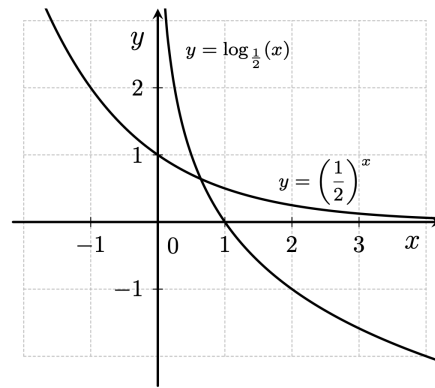
c)  $f(x) = e^x$  et  $g(x) = \ln(x)$

d)  $f(x) = 10^{-x}$  et  $g(x) = \log_{0,1}(x)$

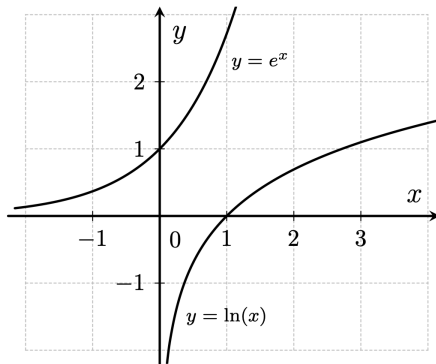
a)



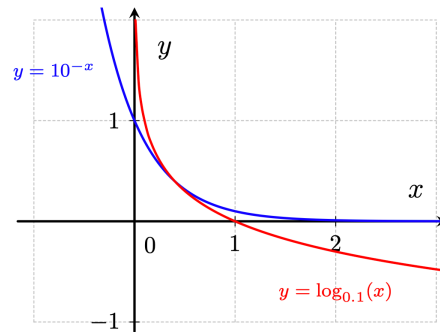
b)



c)



d)



2.2.13 Esquisser le graphe des fonctions données par :

a)  $f(x) = E(2x)$

b)  $f(x) = E\left(\frac{x}{3}\right)$

c)  $f(x) = x \cdot E(x)$

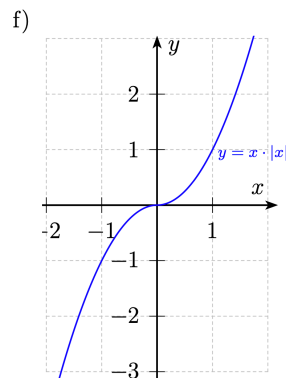
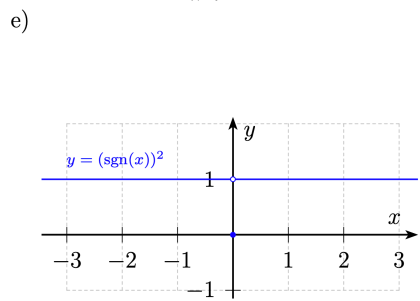
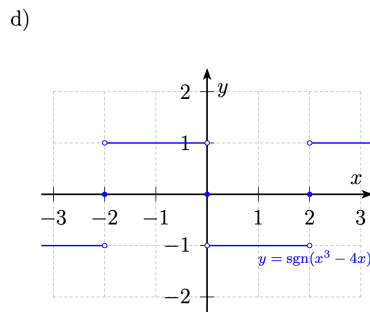
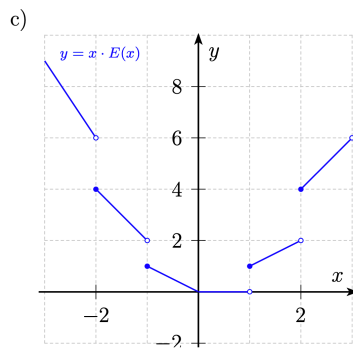
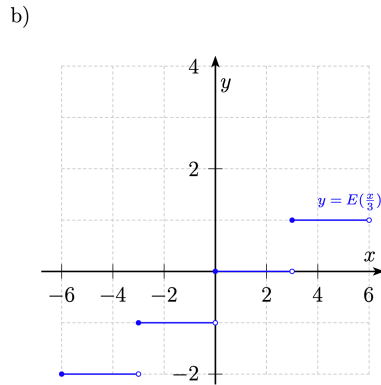
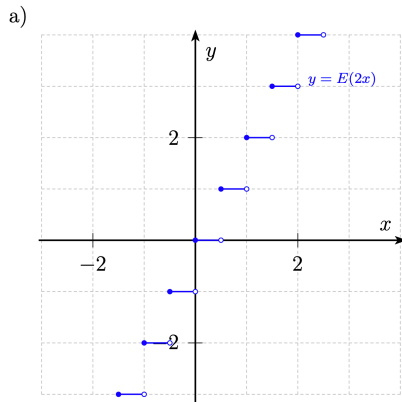
d)  $f(x) = \text{sgn}(x^3 - 4x)$

e)  $f(x) = (\text{sgn}(x))^2$

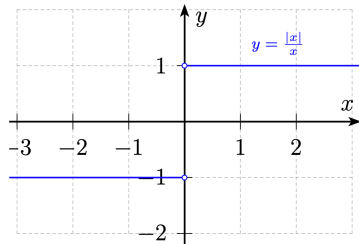
f)  $f(x) = x \cdot |x|$

g)  $f(x) = \frac{|x|}{x}$

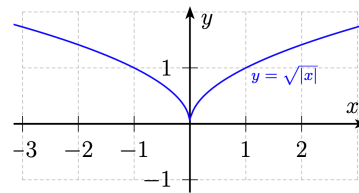
h)  $f(x) = \sqrt{|x|}$



g)



h)



2.2.15 Tracer le graphe des fonctions suivantes :

a)  $f(x) = |x| - 2$ ,  $g(x) = |x| + 1$ ,  $h(x) = |x - 3|$ ,  $k(x) = |x + 1|$  et  $l(x) = -|x| + 1$

b)  $f(x) = \sqrt{x+4}$ ,  $g(x) = 2\sqrt{x}$  et  $h(x) = \sqrt{x-1} - 4$

c)  $f(x) = \frac{1}{x} - 3$  et  $g(x) = \frac{1}{x-2}$

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d)  $f(x) = 2^x - 2$ ,  $g(x) = 2^{x+1}$  et  $h(x) = -2^x$

e)  $f(x) = \ln(x-1)$ ,  $g(x) = 2\ln(x)$ ,  $h(x) = \ln\left(\frac{1}{x}\right)$  et  $k(x) = |\ln(x)|$

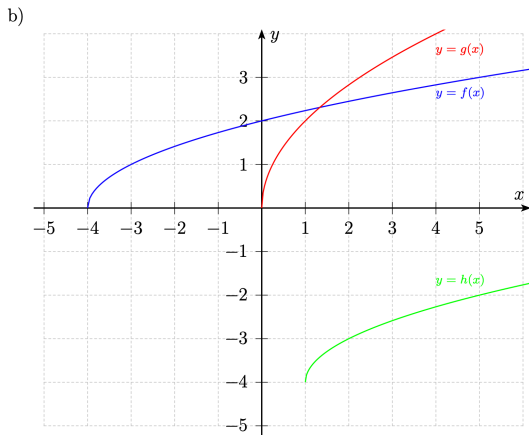
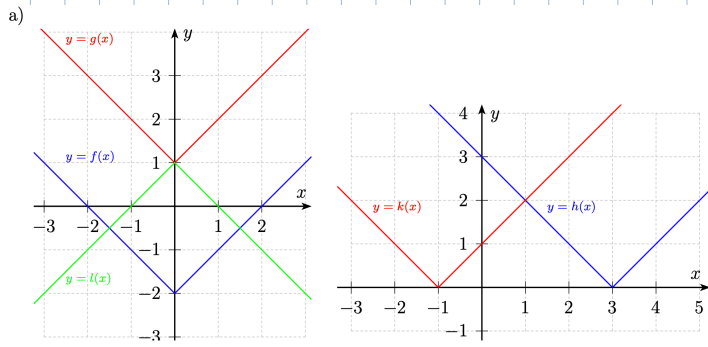
f)  $f(x) = 2 + \sin(x)$  et  $g(x) = |\cos(x)|$

a)  $f(x) = |x| - 2$   $E_{f^{-1}} = \mathbb{R}$

$$f(x) = \begin{cases} x-2 & \text{si } x \geq 0 \\ -x-2 & \text{si } x < 0 \end{cases}$$

$$f(x) = |x-3| = \begin{cases} x-3 & \text{si } x \geq 3 \\ -x+3 & \text{si } x < 3 \end{cases}$$

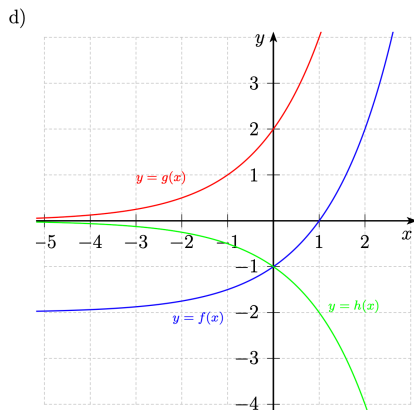
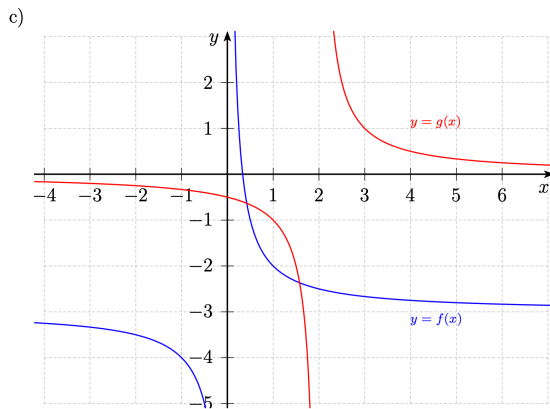
$$f(x) = -|x| + 1 = \begin{cases} -x+1 & \text{si } x \geq 0 \\ x+1 & \text{si } x < 0 \end{cases}$$

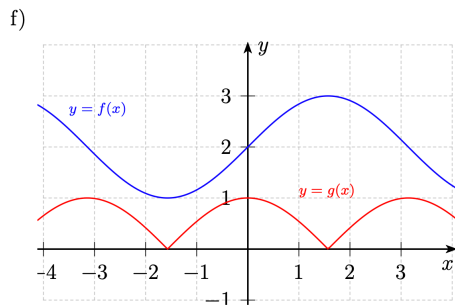
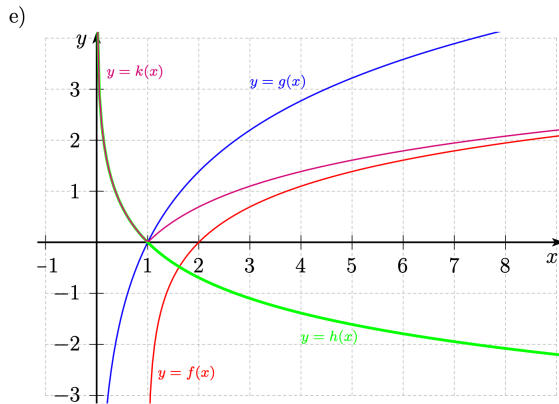


$$y = \sqrt{x+4} \quad x \in [-4; +\infty[$$

$$y \in [0; +\infty[$$

$$\Rightarrow y^2 = x+4 \Rightarrow x = y^2 - 4$$





**2.2.16** Tracer le graphe des fonctions suivantes :

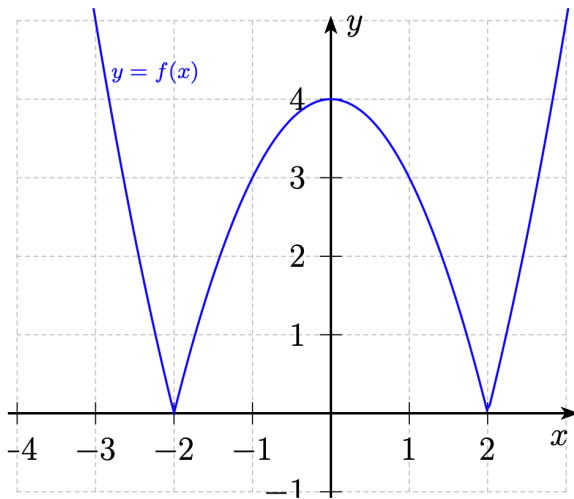
a)  $f(x) = |4 - x^2|$

b)  $f(x) = ||x + 4| - 2| + 1$

c)  $f(x) = |x^2 - 2x| - 1$

d)  $f(x) = |x - 1| + |x + 2|$

a)

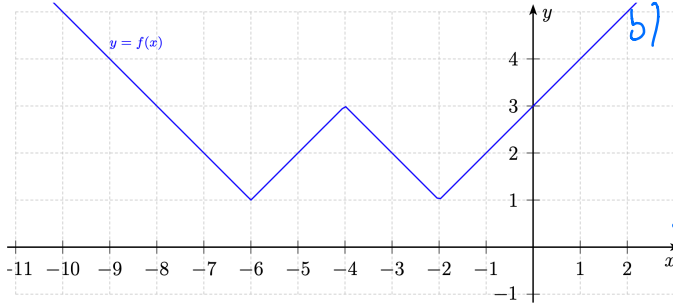


a)  $f(x) = |4 - x^2|$

$x$	$-\infty$	$-2$	$2$	$+\infty$
$-x^2 + 4$		$-$	$+$	$-$



b)



$$f(x) = \left| |x+4| - 2 \right| + 1$$

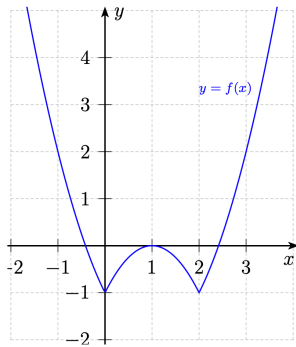
$$|x+4| = \begin{cases} x+4 & \text{si } x \geq -4 \\ -x-4 & \text{si } x < -4 \end{cases}$$

$$|x+4| - 2 = \begin{cases} x+2 & \text{si } x \geq -4 \\ -x-6 & \text{si } x < -4 \end{cases}$$

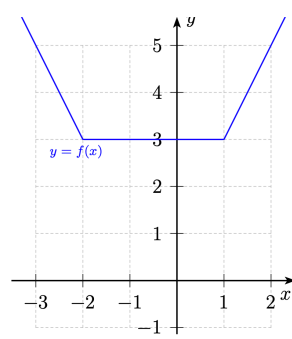
$$||x+4| - 2| = \begin{cases} |x+2| = \begin{cases} x+2 & \text{si } x \geq -2 \\ -x-2 & \text{si } x < -2 \end{cases} \\ |x-6| = \begin{cases} x-6 & \text{si } -6 \leq x < -4 \\ -x-6 & \text{si } x < -6 \end{cases} \end{cases}$$

$$||x+4| - 2| + 1 = \begin{cases} x+3 & \text{si } x \geq -2 \\ -x-1 & \text{si } -4 \leq x < -2 \\ x+7 & \text{si } -6 \leq x < -4 \\ -x-5 & \text{si } x < -6 \end{cases}$$

c)

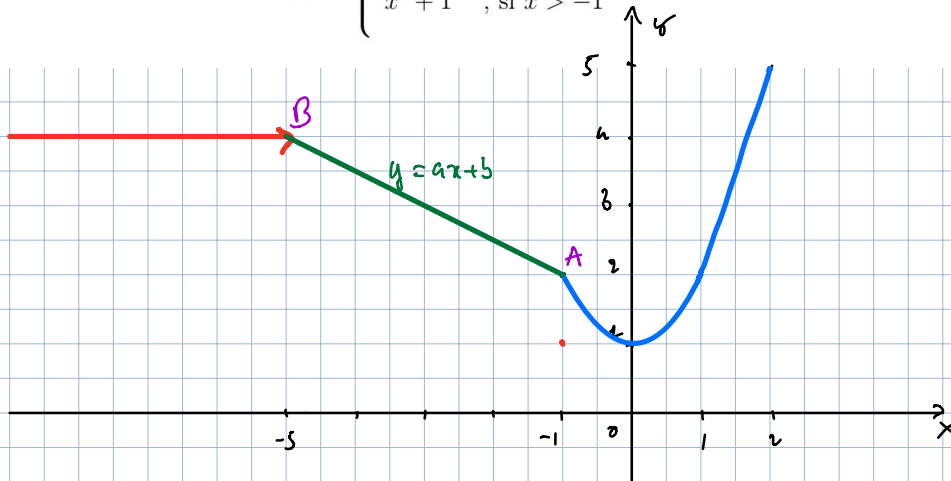


d)



2.2.18 Quelles valeurs doit-on attribuer à  $a$  et à  $b$  pour que le graphe de  $f$  puisse être tracé « sans lever le crayon » ?

$$f(x) = \begin{cases} 4 & , \text{ si } x < -5 \\ ax + b & , \text{ si } -5 \leq x \leq -1 \\ x^2 + 1 & , \text{ si } x > -1 \end{cases}$$



$$B(-5; 4) \text{ et } A(-1; 2)$$

$$y = ax + b$$

$$= \begin{cases} 4 = -5a + b \\ 2 = -a + b \end{cases}$$

$$\Rightarrow \begin{cases} a = -\frac{1}{2} \\ b = \frac{3}{2} \end{cases}$$

2.2.19 Déterminer si les fonctions suivantes sont paires, impaires ou ni l'un ni l'autre :

a)  $f(x) = 9x^4 - 3x^2 + 2$

b)  $f(x) = x^3 - 2x$

c)  $f(x) = 5$

d)  $f(x) = x^2 + 8x + 2$

e)  $f(x) = \frac{3x^2 - 2}{2x}$

f)  $f(x) = \frac{x^5 - x}{x^2 + 1}$

g)  $f(x) = \frac{x}{x+2} + \frac{x}{x-2}$

h)  $f(x) = x^6 + 3x^2 - \frac{1}{x}$

i)  $f(x) = \sqrt{x}$

j)  $f(x) = \sqrt{9 - x^2}$

k)  $f(x) = |x^3 - 3x| + 1$

l)  $f(x) = \frac{x}{|x| - 1}$

m)  $f(x) = \sin(x) + \cos(x)$

n)  $f(x) = \sin^2(x) \cdot \cos(x)$

a)  $f(-x) = 9x^4 - 3x^2 + 2 = f(x) \Rightarrow$   $f(x)$  paire

b)  $f(-x) = (-x)^3 - 2(-x) = -x^3 + 2x = -f(x) \Rightarrow$   $f(x)$  impaire

c)  $f(-x) = 5 = f(x) \Rightarrow$   $f(x)$  paire

d)  $f(-x) = (-x)^2 + 8(-x) + 2 = x^2 - 8x + 2 \neq f(x) \Rightarrow$   $f(x)$  ni paire, ni impaire

e)  $f(-x) = \frac{3(-x)^2 - 2}{2(-x)} = \frac{3x^2 - 2}{-2x} = -f(x) \Rightarrow$   $f(x)$  impaire

f)  $f(-x) = \frac{(-x)^5 - (-x)}{(-x)^2 + 1} = \frac{-x^5 + x}{x^2 + 1} = -f(x) \Rightarrow$   $f(x)$  impaire

g)  $f(-x) = \frac{-x}{-x+2} + \frac{-x}{-x-2} = \frac{-x}{-(x-2)} + \frac{-x}{-(x+2)} = f(x) \Rightarrow$   $f(x)$  paire

h)  $f(-x) = (-x)^6 + 3(-x)^2 - \frac{1}{-x} = x^6 + 3x^2 + \frac{1}{x} \Rightarrow$   $f(x)$  ni paire ni impaire

i)  $f(-x) = \sqrt{-x}$  ! pas définie sur le même domaine  $\Rightarrow$   $f(x)$  ni paire ni impaire

j)  $f(-x) = \sqrt{9 - (-x)^2} = \sqrt{9 - x^2} = f(x) \Rightarrow$   $f(x)$  paire

k)  $f(-x) = |(-x)^3 - 3(-x)| + 1 = |-x^3 + 3x| + 1 = |-(x^3 - 3x)| + 1$   
 $= |x^3 - 3x| + 1 = f(x) \Rightarrow$   $f(x)$  paire

l)  $f(-x) = \frac{-x}{|-x| - 1} = \frac{-x}{|x| - 1} = -\frac{x}{|x| - 1} = -f(x) \Rightarrow$   $f(x)$  impaire

$$m) f(-x) = \sin(-x) + \cos(-x) = -\sin(x) + \cos(x) \neq f(x) \Rightarrow \underline{\text{ni paire ni impaire}}$$

$$n) f(-x) = \sin^2(-x) \cdot \cos(-x) = \sin^2(x) \cos(x) = f(x) \Rightarrow \underline{f(x) \text{ paire}}$$