

Calcul littéral

Exercice 3.1

Réduire les monômes

a) $x \cdot y \cdot z \cdot y \cdot y \cdot z \cdot y \cdot y \cdot y \cdot z \cdot x \cdot z$

e) $-5 \cdot (-12z) \cdot 3z^2y \cdot (-4zy^2)$

b) $(-5x^2) \cdot (2x^3) \cdot (-x^2)$

f) $-4 \cdot (-2x) \cdot (-5y) \cdot (-z)^2$

c) $(-7y^3) \cdot (-2xy) \cdot (-x^2y)$

g) $-6 \cdot (-2x) \cdot (-15x^2) \cdot (-a)^3$

d) $-5xy^3 \cdot (-2x^3y) \cdot (-1)$

h) $\left(\frac{3}{2}x^2y^2z\right) \cdot \left(\frac{2}{3}x^4yz^4\right)$

a) $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z = \underline{x^2 y^6 z^4}$

b) $(-5x^2)(2x^3)(-x^2) = (-5)(2)(-1) \cdot x^2 x^3 x^2 = 10x^{2+3+2} = \underline{10x^7}$

c) $(-7y^3) \cdot (-2xy) \cdot (-x^2y) = (-7)(-2)(-1) y^3 xy x^2 y = \underline{-14x^3y^5}$

d) $-5xy^3 \cdot (-2x^3y) \cdot (-1) = (-5)(-1)(-2) xy^3 x^3 y = \underline{-10x^4y^4}$

e) $-5 \cdot (-12z) \cdot 3z^2y \cdot (-4zy^2) = (-5)(-12) \cdot 3 \cdot (-4) z^2 y z y^2 = \underline{-240y^3z^4}$

f) $-4 \cdot (-2x) \cdot (-5y) \cdot (-z)^2 = 8x(-5y)z^2 = \underline{-40xyz^2}$

g) $-6 \cdot (-2x) \cdot (-15x^2) \cdot (-a)^3 = (-6)(-2)(-15) x x^2 (-a^3) = \underline{180a^3x^3}$

h) $\left(\frac{3}{2}x^2y^2z\right) \cdot \left(\frac{2}{3}x^4yz^4\right) = \frac{3}{2} \cdot \frac{2}{3} x^2 x^4 y^2 y z z^4 = \underline{x^6 y^3 z^5}$

Exercice 3.2

Réduire les monômes

a) $(3xy^2)(z^3yx)(-7x^2y^2z^2)$

c) $(2xyy)(z^3xy)(3x)(2y)(6z^3)$

b) $(-5x^6y^7z^3)(28xyzyz)(2xyz)$

d) $(5x^2y^2)(5x^2y^2)(5x^2y^2)$

$$a) (3xy^2)(z^3yx)(-7x^2y^2z^2) = -21 x x x^2 y^2 y y^2 z^3 z^2 = \underline{-21 x^4 y^5 z^5}$$

$$b) (-5x^6y^7z^3)(28xyzyz)(2xyz) = (-5) \cdot 28 \cdot 2 x^6 y^7 z^3 x^2 y^2 z^2 x y z = \underline{-280 x^9 y^{10} z^6}$$

$$c) (2xyy)(z^3xy)(3x)(2y)(6z^3) = 2 \cdot 3 \cdot 2 \cdot 6 x y^2 z^3 x y x y z^3 = \underline{72 x^3 y^4 z^6}$$

$$d) (5x^2y^2)(5x^2y^2)(5x^2y^2) = (5x^2y^2)^3 = 5^3 x^6 y^6 = \underline{125 x^6 y^6}$$

Exercice 3.3

Réduire les monômes

$$a) 3(xy^2)^3 = 3x^3y^6$$

$$b) (3x^2y)^3 = 3^3x^6y^3 = 27x^6y^3$$

$$c) (-z^2y^2)^2 = (-1)^4z^4y^4 = z^4y^4$$

$$d) -(zy^2)^2 = -1^2z^2y^4 = -z^2y^4$$

$$e) (-xy^2z^3)^3 = (-1)^3x^3y^6z^9 = -x^3y^6z^9$$

$$f) (-x)^{16} = x^{16}$$

$$g) (-x)^{27} = -x^{27}$$

$$h) (-3x^4y^5z^6)^5 = (-1)^53^5x^{20}y^{25}z^{30} = -243x^{20}y^{25}z^{30}$$

Rappel :

$$(-a)^n = \begin{cases} a^n & \text{si } n \text{ est pair} \\ -a^n & \text{si } n \text{ est impair} \end{cases}$$

Exercice 3.4

Réduire les monômes

$$a) a^2 \cdot a^3 \cdot a = a^{2+3+1} = a^6$$

$$b) (a^2)^4 = a^{2 \cdot 4} = a^8$$

$$c) c^5 \cdot c^3 \cdot c^2 = c^{5+3+2} = c^{10}$$

$$d) (a \cdot b^2 \cdot c)^3 = a^3b^6c^3$$

$$e) (a^2)^3 \cdot (a^4)^2 = a^6a^8 = a^{6+8} = a^{14}$$

$$f) x \cdot x = x^2$$

$$g) x + x = 2x$$

$$h) b^5 \div b^3 = \frac{b^5}{b^3} = b^{5-3} = b^2$$

$$i) x + x + x = 3x$$

$$j) x^2 \cdot x^4 \cdot x = x^{2+4+1} = x^7$$

$$k) (a^2 \cdot b^3 \cdot c)^3 = a^6b^9c^3$$

$$l) x + x^2 = x^2 + x$$

$$m) \frac{c}{2} \cdot 20c = \frac{20c^2}{2} = 10c^2$$

$$n) \frac{y^4}{y} = y^{4-1} = y^3$$

$$o) (\sqrt{2}x) \cdot (\sqrt{2}xy) = \sqrt{2}\sqrt{2}xx y = 2x^2y$$

$$p) \frac{1}{3}a^4 \cdot \frac{9}{2}a^3 = \frac{1}{3} \cdot \frac{9}{2} a^{4+3} = \frac{3}{2}a^7$$

Exercice 3.5

Réduire les monômes

a) $(-x)^2 = x^2$

b) $-x^2 = -x^2$

c) $-x^2 + x^2 = 0$

d) $-x^2 - x^2 = -2x^2$

e) $-x^2 \cdot x^2 = -x^{2+2} = -x^4$

f) $-x^2 \cdot (-x^2) = x^4$

g) $-(x^2)^2 = -x^4$

h) $(-x^2)^2 = x^4$

i) $(2xyz)^4 = 2^4 x^4 y^4 z^4 = 16x^4 y^4 z^4$

j) $2(xy)^4 z = 2x^4 y^4 z$

k) $(-2yz)^3 = -8y^3 z^3$

l) $-2^3(yz)^2 = -8y^2 z^2$

m) $\frac{1}{8}(2yz)^3 = \frac{1}{8} 2^3 y^3 z^3 = y^3 z^3$

n) $-3(3z)^2 = -3 \cdot 3^2 z^2 = -27z^2$

o) $(-2xy)^2 = 2^2 x^2 y^2 = 4x^2 y^2$

p) $\frac{1}{5}(5xy)^2 = \frac{1}{5} \cdot 5^2 x^2 y^2 = 5x^2 y^2$

q) $(-y)^5 = -y^5$

r) $(2x^2 y^3)^4 = 2^4 x^{2 \cdot 4} y^{3 \cdot 4} = 16x^8 y^{12}$

s) $\left(\frac{3}{4}x\right)^2 = \frac{3^2}{4^2} x^2 = \frac{9}{16} x^2$

t) $(0,5x)^3 = \left(\frac{1}{2}x\right)^3 = \frac{x^3}{8} = 0,125x^3$

u) $\left(\frac{1}{2}x^2 y\right)^2 = \frac{1}{4} x^4 y^2$

v) $8 \cdot \left(\frac{-5x^3}{2}\right)^2 = 8 \cdot \frac{5^2 x^6}{2^2}$

$= 50x^6$

Exercice 3.6

Quel est le coefficient des monômes ? Quel est le degré des monômes ? Quel est le degré des monômes relativement à x , y et z ?

Monôme	Coefficient	Degré du monôme	Degré en x	Degré en y	Degré en z
a) $2x^2$	2	2	2	0	0
b) $3xy$	3	2	1	1	0
c) $-24y^2x$	-24	3	1	2	0
d) $64x^6y^3$	64	9	6	3	0
e) $\sqrt{2}x^4y^3z^2$	$\sqrt{2}$	9	4	3	2
f) $-6x^4y^4z^4$	-6	12	4	4	4
g) $(-\sqrt{5}y)^2$	5	2	0	2	0
h) $\frac{3z}{10}$	$\frac{3}{10} = 0,3$	1	0	0	1
i) $3x - 4x$	-1	1	1	0	0
j) $3\sqrt{3}$	$3\sqrt{3}$	0	0	0	0
k) $(2xy)^3$	8	6	3	3	0
l) $-12(xy)^5z^3$	-12	13	5	5	3

Exercice 3.7

Associer les monômes semblables

Rappel: Deux monômes sont dits semblables s'ils ont la même partie littérale

- a)
- | | | | |
|-----------------|--------|------------|---------------|
| $4x$ | $4x^2$ | $3x^3$ | $\sqrt{3}x^2$ |
| $0,5x^2$ | $-8x$ | $(-2x)^4$ | $4x^4$ |
| $\frac{x^4}{2}$ | x | $-x^2$ | $(-2x)^2$ |
| 4^4 | x^4 | $(2x^2)^2$ | $\frac{x}{5}$ |
- b)
- | | | | |
|----------------|--------------------|--------------------------|------------------------------|
| $2ab$ | $-ba$ | $(ab)^2$ | aab |
| a^2b | $\frac{1}{4}ab^2$ | $(\sqrt{3} + \sqrt{5})a$ | $\left(\frac{a}{5}\right)^4$ |
| a^4 | $\sqrt{2}ab$ | $\frac{-a^2b}{4}$ | $-ab^2$ |
| $\frac{ab}{2}$ | $\frac{a^2b^2}{8}$ | $2b^2$ | $(0,5a^2c)^2$ |

Exercice 3.8

Associer les monômes semblables.

- | | | | |
|--------------|---------------------|--------------------|---------------|
| $3x$ | $\frac{x^2}{2}$ | $\frac{x}{3}$ | $7x \cdot 3y$ |
| $x \cdot 3x$ | | | $(0,5x)^2$ |
| $-x$ | $2 \cdot 2x^2$ | $-2x^2y$ | |
| $\sqrt{6}y$ | $\frac{y}{4}(2x)^2$ | $4 \cdot 3xy$ | |
| $-5xy$ | $(xy)^2$ | $\frac{y}{3}(-3x)$ | |

Exercice 3.9

Monômes

Cette maison est un cube de côté égal à $4x$.

- Calculer, en fonction de x , l'aire latérale de cette maison.
- Calculer, en fonction de x , le volume de cette maison.



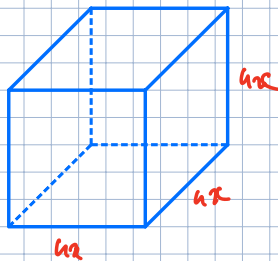
Rappel :

Une aire latérale d'un solide est la surface délimitant ce solide privée de sa (ou ses) base(s).

a) Aire d'une face : $4x \cdot 4x = \underline{16x^2}$

$\Rightarrow$ Aire latérale = Aire des 4 faces : $4 \cdot 16x^2 = \underline{64x^2}$

b)



Volume : $4x \cdot 4x \cdot 4x = (4x)^3 = \underline{64x^3}$

Exercice 3.10

Monômes

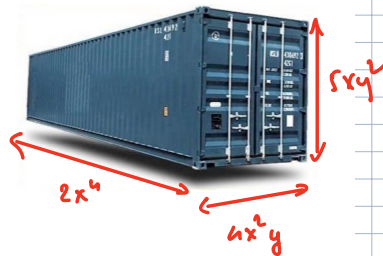
Ce conteneur est un parallélépipède rectangle dont les côtés (en cm) sont égaux à $2x^4$, $4x^2y$ et $5xy^2$.

a) Calculer, en fonction de x et y , l'aire totale de ce conteneur.

b) Calculer, en fonction de x et y , le volume de ce conteneur.

c) Quelles sont les dimensions du conteneur si $x = 3$ et $y = 5$?

d) Quel est le volume du conteneur si $x = 3$ et $y = 5$?



a) Fond : $2x^4 \cdot 4x^2y = 8x^6y$

Avant : $4x^2y \cdot 5xy^2 = 20x^3y^3$

Côté : $2x^4 \cdot 5xy^2 = 10x^5y^2$

$\Rightarrow$ Aire totale : $2 \cdot 8x^6y + 2 \cdot 20x^3y^3 + 2 \cdot 10x^5y^2 = \underline{16x^6y + 40x^3y^3 + 20x^5y^2} \text{ [cm}^2\text{]}$

b) $V = 2x^4 \cdot 4x^2y \cdot 5xy^2 = \underline{40x^6y^3} \text{ [cm}^3\text{]}$

c) si $x = 3$, $y = 5$

$\Rightarrow 2x^4 = 2 \cdot 3^4 = 162 \text{ cm}$

$4x^2y = 4 \cdot 3^2 \cdot 5 = 180 \text{ cm}$

$5xy^2 = 5 \cdot 3 \cdot 5^2 = 375 \text{ cm}$

$\Rightarrow$ Les dimensions sont : $162 \times 180 \times 375 \text{ cm}$

d) $V = 162 \cdot 180 \cdot 375 = 10'935'000 \text{ cm}^3 = 10'935 \text{ dm}^3 = \underline{10,935 \text{ m}^3}$

Exercice 3.11

Somme de monômes

$$\text{a) } 7x + 9x - 10x - 20x = (7 + 9 - 10 - 20)x = \underline{-14x}$$

$$\text{b) } -14y^2 - 9y^2 + 12y^2 + 10y^2 = (-14 - 9 + 12 + 10)y^2 = \underline{-y^2}$$

$$\text{c) } -4xy^2 + 29xy^2 - 27xy^2 + 61xy^2 - 18xy^2 = (-4 + 29 - 27 + 61 - 18)xy^2 = \underline{41xy^2}$$

$$\text{d) } 15x^2y^2 + (-19x^2y^2) + (-11x^2y^2) + 14x^2y^2 + (-29x^2y^2) = (15 - 19 - 11 + 14 - 29)x^2y^2 = \underline{-30x^2y^2}$$

$$\text{e) } 14xz^2 - 3xz^2 + 15xz^2 + 17xz^2 - 24xz^2 = (14 - 3 + 15 + 17 - 24)xz^2 = \underline{19xz^2}$$

$$\text{f) } 24x^2yz^2 + 12x^2yz^2 - 18x^2yz^2 - 5x^2yz^2 = (24 + 12 - 18 - 5)x^2yz^2 = \underline{13x^2yz^2}$$

Exercice 3.12

Somme de monômes

$$\text{a) } 5x + (\underbrace{x + 3x}_{4x}) = 5x + 4x = \underline{9x}$$

$$\text{b) } 8x - (\underbrace{24x - 5x}_{19x}) = 8x - 19x = \underline{-11x}$$

$$\text{c) } 12y - (-8y + 7y - 3y) = 12y - (-4y) = 12y + 4y = \underline{16y}$$

$$\text{d) } x - [3x - (2x - 5x)] = x - [3x - (-3x)] = x - [3x + 3x] = x - 6x = \underline{-5x}$$

$$\text{e) } 3x - \{2x - [4x - (5x - 8x)]\} = 3x - \{2x - [4x - (-3x)]\} = 3x - \{2x - [4x + 3x]\} = 3x - \{2x - 7x\} = 3x - (-5x) = 3x + 5x = \underline{8x}$$

$$\begin{aligned} \text{f) } 24x^2 - \{7x + [3x - (8x + 2x)]\} \\ = 24x^2 - \{7x + [3x - 10x]\} \\ = 24x^2 - \{7x + [-7x]\} = 24x^2 - \{7x - 7x\} = \underline{24x^2} \end{aligned}$$

e) on peut aussi faire : $3x - 2x + 4x - 5x + 8x = (3 - 2 + 4 - 5 + 8)x = 8x$

Exercice 3.13

Somme de polynômes

$$\text{a) } 4x^2 - 5 + 3x^2 - 1 = \underline{7x^2 - 6}$$

$$\text{b) } 6 + x^3 + 2 - x^3 = \underline{8}$$

$$\text{c) } 5x^2 + 9y^2 - 4y^2 + x^2 = \underline{6x^2 + 5y^2}$$

$$\text{d) } 6x^2 - 5xy + 5x^2 - 6xy = \underline{11x^2 - 11xy}$$

$$\text{e) } 4xyz - y^2 + y^2 - 4xyz = \cancel{4xyz} - \cancel{4xyz} - \cancel{y^2} + \cancel{y^2} = \underline{0}$$

$$\text{f) } xy + 17yz + 5xy - 1 = \underline{6xy + 17yz - 1}$$

Exercice 3.14

Somme de polynômes

$$\text{a) } (4x - 5) + (3x - 1) = 4x - 5 + 3x - 1 = 4x + 3x - 5 - 1 = \underline{7x - 6}$$

$$\text{b) } (5x + 9y) + (-4y + x) = 5x + 9y - 4y + x = 5x + x + 9y - 4y = \underline{6x + 5y}$$

$$\text{c) } (6 - 5x) + (5x - 6) = \cancel{6} - \cancel{5x} + \cancel{5x} - \cancel{6} = \underline{0}$$

$$\text{d) } 6x^2 + (-5x^2 + 1) = 6x^2 - 5x^2 + 1 = \underline{x^2 + 1}$$

$$\text{e) } (15x + 3y - z) + (-14x + 2y + z) = 15x + 3y - z - 14x + 2y + z = \underline{x + 5y}$$

$$\text{f) } (20x - 5y - 8z) + (5y + 7z - 19x) = 20x - 5y - 8z + 5y + 7z - 19x = \underline{x - z}$$

$$\text{g) } (x^3 - 5x^2 + 7) + (3x + x^3 - 1) = x^3 - 5x^2 + 7 + 3x + x^3 - 1 = \underline{2x^3 - 5x^2 + 3x + 6}$$

$$\begin{aligned} \text{h) } (4x^4 - 5x^3 + 6x^2 - 1) + (3x^4 + 4x^2 - 5) &= 4x^4 - 5x^3 + 6x^2 - 1 + 3x^4 + 4x^2 - 5 \\ &= \underline{7x^4 - 5x^3 + 10x^2 - 6} \end{aligned}$$

Exercise 3.15

Différence de polynômes

$$\text{a) } (5x + y) - (5x + y) = \cancel{5x} + \cancel{y} - \cancel{5x} - \cancel{y} = \underline{0}$$

$$\text{b) } (2x - 5) - (x + 5) = \cancel{2x} - 5 - \cancel{x} - 5 = \underline{x - 10}$$

$$\text{c) } (5x + 3y) - (7x + 3y + z) = \cancel{5x} + \cancel{3y} - \cancel{7x} - \cancel{3y} - z = \underline{-2x - z}$$

$$\text{d) } (6x^2 - 4x) - (6x^2 - 10x) = \cancel{6x^2} - 4x - \cancel{6x^2} + 10x = \underline{6x}$$

$$\text{e) } (8x + 3y - z) - (5x + 3y + z) = \cancel{8x} + \cancel{3y} - z - \cancel{5x} - \cancel{3y} - z = \underline{3x - 2z}$$

$$\text{f) } (8x + 3y - 2z) - (5x - 2y - z) = \cancel{8x} + \cancel{3y} - 2z - \cancel{5x} + \cancel{2y} + z = \underline{3x + 5y - z}$$

$$\text{g) } (-19x - 5y + 1) - (4x - 2y - 3) = \cancel{-19x} - 5y + 1 - \cancel{4x} + \cancel{2y} + 3 = \underline{-23x - 3y + 4}$$

$$\begin{aligned} \text{h) } (4x^4 + 2x^3 + x^2) - (5x^3 + 5x^2 + x - 7) &= \cancel{4x^4} + \cancel{2x^3} + \cancel{x^2} - 5x^3 - 5x^2 - x + 7 \\ &= \underline{\cancel{4x^4} - 3x^3 - 4x^2 - x + 7} \end{aligned}$$

Exercise 3.16

Calcul polynomial

$$\text{a) } 16x - y - (3x - y) + (4x - 1) - (x + y) = \cancel{16x} - \cancel{y} - \cancel{3x} + \cancel{y} + \cancel{4x} - 1 - \cancel{x} - y = \underline{16x - y - 1}$$

$$\text{b) } 17y - (5y - 3) - (15y + 1) + (3y - 2) - 1 = \cancel{17y} - \cancel{5y} + 3 - \cancel{15y} - 1 + \cancel{3y} - 2 - 1 = \underline{-1}$$

$$\begin{aligned} \text{c) } 20x - [16x - (2x + y)] &= 20x - [16x - 2x - y] = 20x - [14x - y] = 20x - 14x + y \\ &= \underline{6x + y} \end{aligned}$$

$$\text{d) } 15x - [5x - (2x - y)] = 15x - [5x - 2x + y] = 15x - 3x - y = \underline{12x - y}$$

$$\text{e) } 6x^2 - [15x - (3x^2 - 9x)] = 6x^2 - [15x - 3x^2 + 9x] = 6x^2 - 24x + 3x^2 = \underline{9x^2 - 24x}$$

$$\text{f) } 187y - 52z - [93y - (76z + 34)] = \cancel{187y} - 52z - \cancel{93y} + 76z + 34 = \underline{94y + 24z + 34}$$

Exercice 3.17

Calcul polynomial

$$a) (4x^2 - 3xy + 8y^2) + (-3x^2 + 3xy - 9y^2) = \cancel{4x^2} - \cancel{3xy} + \cancel{8y^2} - \cancel{3x^2} + \cancel{3xy} - \cancel{9y^2} = x^2 - y^2$$

$$b) (-x + 2y - z) + (2x - y - z) + (-x - y + 2z) = \cancel{-x} + \cancel{2y} - \cancel{z} + \cancel{2x} - \cancel{y} - \cancel{z} - \cancel{x} - \cancel{y} + \cancel{2z} = 0$$

$$c) 12x - 4y - (6x - 4y) - [(3x + 4y) - (2x + 9y)] = \cancel{12x} - \cancel{4y} - \cancel{6x} + \cancel{4y} - [\cancel{3x} + \cancel{4y} - \cancel{2x} - \cancel{9y}] = 6x - [x - 5y] = 5x + 5y$$

$$d) 108y - (24y + 15) - [76y - (15y + 23) - (54y - 7)]$$

$$= 108y - 24y - 15 - [76y - 15y - 23 - 54y + 7]$$

$$= 84y - 15 - [7y - 16] = 84y - 15 - 7y + 16 = 77y + 1$$

Exercice 3.18

Produit de polynômes

$$a) 4(x + 1) = 4x + 4$$

$$e) -1(-x - y) = x + y$$

$$b) 3(x - y) = 3x - 3y$$

$$f) (2x + 3y - 5z)(-1) = -2x - 3y + 5z$$

$$c) -1(x - y) = -x + y$$

$$g) (-1)(3x^2 - 2x - 5) = -3x^2 + 2x + 5$$

$$d) -1(-x + y) = x - y$$

$$h) (-1)[(-1)(-x^2 + y^2)] = -x^2 + y^2$$

Exercice 3.19

Produit de polynômes

$$a) x(x^2 + 2x - 1) = x^3 + 2x^2 - x$$

$$b) 2y(2x + 5) = 4xy + 10y$$

$$c) (-3z + 8)21x^2 = -63x^2z + 168x^2$$

$$d) -x^2(2x + 5x^2 - x^3) = -2x^3 - 5x^4 + x^5 = x^5 - 5x^4 - 2x^3$$

$$e) (-4xz^3 - 5x^3z)(-xz^3) = 4x^2z^6 + 5x^4z^4 = 5x^4z^4 + 4x^2z^6$$

$$f) -3xy^2z^3(2x^3y^2z - 6x^2y + 4x) = -6x^4y^4z^3 + 18x^3y^4z^3 - 12x^2y^4z^3 = -6x^4y^4z^3 + 18x^3y^4z^3 - 12x^2y^4z^3$$

Exercice 3.20

Réduire les produits de polynômes

$$a) (2x - 5)(3y - 7) = \underline{6xy - 14x - 15y + 35}$$

$$b) (9z - 11)(12 - 15y) = 108z - 135yz - 132 + 165y = \underline{165y - 135yz + 108z - 132}$$

$$c) (15x - 1)(2z + 12) = \underline{30xz + 180x - 2z - 12}$$

$$d) (-x + 1)(-y - 1) = \underline{xy + x - y - 1}$$

$$e) (-3x - 2y)(-x - y) = 3x^2 + 3xy + 2xy + 2y^2 = \underline{3x^2 + 5xy + 2y^2}$$

$$f) (x^2 - 1)(x + 1) = \underline{x^3 + x^2 - x - 1}$$

$$g) (y^2 - 1)(y^4 + 1) = y^6 + y^2 - y^4 - 1 = \underline{y^6 - y^4 + y^2 - 1}$$

$$h) (10x^2 - 5y)(2x - 6y^2) = \underline{20x^3 - 60x^2y^2 - 10xy + 30y^3}$$

Exercice 3.21

Réduire les produits de polynômes

$$a) (x^2 - 2x)(3x - 1) = 3x^3 - x^2 - 6x^2 + 2x = \underline{3x^3 - 7x^2 + 2x}$$

$$b) (5y - 1)(y^2 + 2y) = 5y^3 + 10y^2 - y^2 - 2y = \underline{5y^3 + 9y^2 - 2y}$$

$$c) (3y - 1)(4y^2 - 5y + 6) = 12y^3 - 15y^2 + 18y - 4y^2 + 5y - 6 = \underline{12y^3 - 19y^2 + 23y - 6}$$

$$d) (6z^2 - 3z - 4)(5z - 3) = 30z^3 - 18z^2 - 15z^2 + 9z - 20z + 12 = \underline{30z^3 - 33z^2 - 11z + 12}$$

$$e) (x + 1)(x^2 - x - 1) = x^3 - x^2 - x + x^2 - x - 1 = \underline{x^3 - 2x - 1}$$

$$f) (x^2 + x - 3)(x^2 - 2x + 1) = x^4 - 2x^3 + x^2 + x^3 - 2x^2 + x - 5x^2 + 6x - 3 = \underline{x^4 - x^3 - 4x^2 + 7x - 3}$$

$$g) (x - y)(x + y)(x - z) = (x^2 - y^2)(x - z) = x^3 - x^2z - xy^2 + y^2z$$

$$h) (2x + 1)(3x - 1)(5x + 2) = (6x^2 - 2x + 3x - 1)(5x + 2) = 30x^3 + 12x^2 - 10x^2 - 4x + 15x^2 + 6x - 5x - 2 = \underline{30x^3 + 17x^2 - 5x - 2}$$

Exercice 3.22

Réduire les produits de polynômes

$$a) (x^2 + 2x + 1)(x + 1) = x^3 + x^2 + 2x^2 + 2x + x + 1 = \underline{x^3 + 3x^2 + 3x + 1}$$

$$b) 3x(x - y)(x + 2y) = (3x^2 - 3xy)(x + 2y) = 3x^3 + 6x^2y - 3x^2y - 6xy^2 = \underline{3x^3 + 3x^2y - 6xy^2}$$

$$c) -5xy(2x + 3y)(4x + 1)$$

$$d) (x + z)(x - 2z)(2x - z)$$

$$e) (2 - x)(3 + x)(7x + 1)$$

$$f) (2n + 1)(3n - 1)(5n + 2)$$

$$c) -5xy(2x + 3y)(4x + 1) = (-10x^2y - 15xy^2)(4x + 1) \\ = \underline{-40x^3y - 10x^2y - 60xy^2 - 15xy^2}$$

$$d) (x + z)(x - 2z)(2x - z) = (x^2 - 2xz + 2z - 2z^2)(2x - z) \\ = (x^2 - 2z - 2z^2)(2x - z) = 2x^3 - x^2z - 2x^2z + xz^2 - 4xz^2 + 2z^3 \\ = \underline{2x^3 - 3x^2z - 3xz^2 + 2z^3}$$

$$e) (2 - x)(3 + x)(7x + 1) = (6 + 2x - 3x - x^2)(7x + 1) \\ = (6 - x - x^2)(7x + 1) = 42x + 6 - 7x^2 - x - 7x^3 - x^2 \\ = \underline{-7x^3 - 8x^2 + 41x + 6}$$

$$f) (2y + 1)(3y - 1)(5y + 2) = (6y^2 - 2y + 3y - 1)(5y + 2) \\ = (6y^2 + y - 1)(5y + 2) = 30y^3 + 12y^2 + 5y^2 + 2y - 5y - 2 \\ = \underline{30y^3 + 17y^2 - 3y - 2}$$

Exercice 3.23

Calcul polynomial

On donne :

$$A = 5a^3 + 3a^2b - 7b^3, \quad B = 8a^3 - 3b^3 - 9a^2b,$$

$$C = 9a^2b - 3a^3 - 7b^3, \quad D = 8a^2b - 2b^3 - 7a^3$$

Calculer :

a) $X = A - [B - (C - D)]$

b) $Y = C - \{B - [D - (A + C)]\}$

$$a) X = A - [B - (C - D)] = A - [B - C + D] = A - B + C - D$$

$$\Rightarrow X = 5a^3 + 3a^2b - 7b^3 - (8a^3 - 3b^3 - 9a^2b) + 9a^2b - 3a^3 - 7b^3 - (8a^2b - 2b^3 - 7a^3)$$

$$\Rightarrow X = 5a^3 + 3a^2b - 7b^3 - 8a^3 + 3b^3 + 9a^2b + 9a^2b - 3a^3 - 7b^3 - 8a^2b + 2b^3 + 7a^3$$

$$\Rightarrow X = a^3 - 9b^3 + 13a^2b = \underline{a^3 + 13a^2b - 9b^3}$$

$$b) Y = C - \{B - [D - (A + C)]\} = C - \{B - D + (A + C)\}$$

$$\Rightarrow Y = C - B + D - (A + C) = \cancel{C} - B + D - A - \cancel{C} = -B + D - A$$

$$\Rightarrow Y = -(8a^3 - 3b^3 - 9a^2b) + 8a^2b - 2b^3 - 7a^3 - (5a^3 + 3a^2b - 7b^3)$$

$$\Rightarrow Y = -8a^3 + 3b^3 + 9a^2b + 8a^2b - 2b^3 - 7a^3 - 5a^3 - 3a^2b + 7b^3$$

$$\Rightarrow Y = -20a^3 + 8b^3 + 14a^2b = \underline{-20a^3 + 14a^2b + 8b^3}$$

Exercice 3.24

Placer dans le un monôme afin que l'égalité soit vérifiée.

a) $x^3 \cdot \boxed{x^2} = x^4 \cdot x = x^5$

c) $5k^{12}x^3 \cdot \boxed{1/5} \cdot 2kx = 2k^{13}x^4$

b) $4m^3n^2 \cdot 6m^4 = \boxed{3m^6n^2} \cdot 8mn$
 $24m^3n^2$

d) $(x^2y)^3 \cdot \left(\boxed{xy^3} \right)^2 = (xy)^8 \cdot y = x^8y^9$
 x^6y^3 x^2y^6

Exercice 3.25

Calcul polynomial

a) $\frac{4}{3}x^2y - x^2y + \frac{5}{2}x^2y = x^2y \left(\frac{4}{3} - 1 + \frac{5}{2} \right) = x^2y \left(\frac{8-6+5}{6} \right) = \underline{\underline{\frac{17}{6}x^2y}}$

b) $\frac{5}{6}a^3 - a \left(\frac{1}{2}a^2 - \frac{5}{4}ab \right) + \frac{3}{4}a^2b = \frac{5}{6}a^3 - \frac{1}{2}a^3 + \frac{5}{4}a^2b + \frac{3}{4}a^2b$

$= \frac{5a^3 - 3a^3}{6} + \frac{8a^2b}{4} = \frac{2a^3}{6} + 2a^2b = \underline{\underline{\frac{a^3}{3} + 2a^2b}}$

c) $\left(\frac{x^2}{2} - 2 \right) \left(\frac{x}{3} - 1 \right) = \frac{x^3}{6} - \frac{x^2}{2} - \frac{2x}{3} + 2$

d) $\left(\frac{x^3}{2} - \frac{x^2}{4} \right) - \left[\frac{x}{3} \left(\frac{x^2}{2} - \frac{3x}{4} \right) \right] = \frac{x^3}{2} - \frac{x^2}{4} - \frac{x^3}{6} + \frac{x^2}{4} = \frac{3x^3 - x^3}{6} = \frac{2x^3}{6} = \underline{\underline{\frac{x^3}{3}}}$

e) $\frac{x^2 - x}{4} \cdot \frac{2x^2 + 3x}{3} = \frac{2x^4 + 3x^3 - 2x^3 - 3x^2}{12} = \frac{2x^4 + x^3 - 3x^2}{12} = \underline{\underline{\frac{x^4}{6} + \frac{x^3}{12} - \frac{x^2}{4}}}$

f) $\frac{x^2}{8} \left(\frac{8x}{3} - 1 \right) - \frac{3x}{2} \left(\frac{x^2}{6} - \frac{2x}{3} - 2 \right) = \frac{x^3 - x^2}{8} - \frac{x^3}{8} + \frac{3x^2}{4} + 3x$

$= \frac{x^3}{8} - \frac{x^2}{8} - \frac{x^3}{8} + \frac{3x^2}{4} + 3x$

$= \frac{x^3 - 2x^3}{8} - \frac{x^2 + 6x^2}{8} + 3x = \underline{\underline{-\frac{x^3}{4} - \frac{7x^2}{8} + 3x}}$

Exercise 3.26

a) $(6x - 5y)(3x + 4y) - (9x + 2y)(2x - 3y) - 2y(7y - 16x) =$

b) $(6x \cdot 5y)(3x \cdot 4y) - (9x + 2y)(2x \cdot 3y) - 2y(7y \cdot 16x) =$

c) $(8x - 3y)(2x \cdot 4y) - (8x \cdot 3y) \cdot (2x \cdot 4y) =$

d) $-(xy - y^2) + x(x - y)y - y^2 - [xy(xy) - (x - y)(x + y)] =$

a) $(6x - 5y)(3x + 4y) - (9x + 2y)(2x - 3y) - 2y(7y - 16x)$

$$= 18x^2 + 24xy - 15xy - 20y^2 - [18x^2 - 27xy + 4xy - 6y^2] - 14y^2 + 32xy$$

$$= \cancel{18x^2} + 9xy - \cancel{20y^2} - \cancel{18x^2} + 23xy + 6y^2 - 14y^2 + 32xy$$

$$= \underline{-28y^2 + 64xy}$$

b) $(6x \cdot 5y)(3x \cdot 4y) - (9x + 2y)(2x \cdot 3y) - 2y(7y \cdot 16x)$

$$= 30xy \cdot 12xy - (9x + 2y)(6xy) - 2y(112xy)$$

$$= 360x^2y^2 - 54x^2y - 12xy^2 - 224xy^2$$

$$= \underline{360x^2y^2 - 54x^2y - 236xy^2}$$

c) $(8x - 3y)(2x \cdot 4y) - (8x \cdot 3y)(2x \cdot 4y) = (8x - 3y) \cdot 8xy - 24xy \cdot 8xy$

$$= 64x^2y - 24xy^2 - 192x^2y^2 = \underline{-192x^2y^2 + 64x^2y - 24xy^2}$$

$$d) -(xy - y^2) + \underbrace{x(x-y)}_{(x^2 - xy) \cdot y} y - y^2 - [\underbrace{xy(x+y)}_{x^2 y^2} - \underbrace{(x-y)(x+y)}_{x^2 - y^2}]$$

$$= -xy + \cancel{y} + x^2 y - xy^2 - \cancel{y} - x^2 y^2 - x^2 - y^2$$

$$= \underline{-x^2 y^2 + x^2 y - xy^2 - xy + x^2 - y^2}$$

Exercise 3.28

Calculus

$$a) \frac{1}{x} + \frac{1}{x} = \frac{1+1}{x} = \frac{2}{x}$$

$$b) \frac{1}{x} + \frac{2}{x} = \frac{1+2}{x} = \frac{3}{x}$$

$$c) \frac{3}{x} - \frac{2}{x} = \frac{3-2}{x} = \frac{1}{x}$$

$$d) \frac{1}{x} - \frac{2}{x} = \frac{1-2}{x} = -\frac{1}{x}$$

$$e) 3 \cdot \frac{1}{x} = \frac{3}{x}$$

$$f) \frac{1}{x} \div 2 = \frac{\frac{1}{x}}{\frac{2}{1}} = \frac{1}{x} \cdot \frac{1}{2} = \frac{1}{2x}$$

$$g) \frac{1}{x} - \frac{1}{2x} = \frac{2-1}{2x} = \frac{1}{2x}$$

$$h) \frac{1}{3x} - \frac{1}{2x} = \frac{2-3}{6x} = -\frac{1}{6x}$$

$$i) \frac{1}{3x} + \frac{1}{2x} = \frac{2+3}{6x} = \frac{5}{6x}$$

$$j) \frac{1}{x} \cdot \frac{1}{x} = \frac{1}{x^2}$$

$$k) \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$$

$$l) \frac{1}{x} + \frac{1}{x+1} = \frac{(x+1)+x}{x(x+1)} = \frac{2x+1}{x(x+1)} = \frac{2x+1}{x^2+x}$$

$$m) 1 + \frac{1}{x} = \frac{x+1}{x}$$

$$n) \frac{1}{x} \div \frac{1}{x^2} = \frac{\frac{1}{x}}{\frac{1}{x^2}} = \frac{1}{x} \cdot \frac{x^2}{1} = x$$

$$o) \frac{1}{x} - \frac{x}{x+1} = \frac{x+1-x^2}{x(x+1)} = \frac{x^2+x+1}{x^2+x}$$

$$p) \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$$

$$q) \frac{1}{x} \cdot \frac{1}{y} = \frac{1}{xy}$$

$$r) \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$$

$$s) \frac{1}{x} \div \frac{1}{y} = \frac{1}{x} \cdot \frac{y}{1} = \frac{y}{x}$$

$$t) \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$$

$$u) \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$$